A STUDY OF THE FATIGUE PROPERTIES OF LIGHTWEIGHT AGGREGATE CONCRETE

SEPTEMBER 1960

NO. 14

by

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Joint Highway Research Project

PURDUE UNIVERSITY
LAFAYETTE INDIANA

Final Report

A STUDY OF THE FATIGUE PROPERTIES

OF LIGHTWEIGHT AGGREGATE CONCRETE

TO: K. B. Woods, Director
Joint Highway Research Project

September 13, 1960

FROM: H. L. Michael, Assistant Director
Joint Highway Research Project

File: 7-4-7
Project: C-36-56G

Attached is a final report entitled, "A Study of the Fatigue Properties of Lightweight Aggregate Concrete". This report was prepared by Mr. W. H. Gray under the direction of Professor J. F. McLaughlin. Mr. Gray also used this report as his thesis for the MSCE degree.

The report is presented to the Board for the record.

Respectfully submitted,

H. L. Michael

Harold L. Michael, Secretary

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Final Report

A STUDY OF THE FATIGUE PROPERTIES OF LIGHTWEIGHT AGGREGATE CONCRETE

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September 21, 1960

ACKNOWLEDGMENTS

This investigation was sponsored by the Joint Highway Research Project at Purdue University under the direction of Professor K. B. Woods. The writer is grateful to this organization for providing the necessary financial support.

The writer wishes to express his sincere appreciation to Professor J. F. McLaughlin, Research Engineer, Joint Highway Research Project, for his helpful suggestions made during the course of the research and the preparation of this thesis.

Thanks are also extended to Professors A.D.M. Lewis and I. W. Burr for their assistance during the preparation of this thesis. Professor Lewis offered much advice on the operation of the Krouse-Purdue and Amsler fatigue machines. Professor Burr's suggestions on the statistical analysis of the data collected in this study were sincerely appreciated.

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ABSTRACT

Gray, Warren H., M.S.C.E., Purdue University, August, 1960. A Study of the Fatigue Properties of Lightweight Aggregate Concrete. Major Professor: John F. McLaughlin.

It was the purpose of this study to determine the fatigue properties of lightweight aggregate concrete and to investigate the effects of varying the mix proportions and strength of the mix on this property. Fatigue tests were performed on concretes having two mix designs; one being designed for a static compressive strength of 3,500 psi and the other designed for a static compressive strength of 6,000 psi. Five batches of concrete were made using each mix design, and from each batch 30 cylindrical specimens 3 inches in diameter and 6 inches in length were cast. About one-half of these specimens were used in this study. Nearly 70 specimens were tested in static compression to arrive at estimates of the static ultimate compressive strengths of the batches of concrete and nearly 50 others were tested in fatigue. Fatigue tests were conducted at various stress levels at speeds of 1,000 and 500 cycles per minute in two different fatigue testing machines. The Krouse-Purdue machine was used to test at a speed of 1,000 cycles per minute and an Amsler machine was used to test at a speed of 500 cycles per minute.

It was found that within the limits of this investigation the fatigue properties of the lightweight aggregate concrete were not

changed by varying the strength of the concrete or aggregate proportions of the concrete. It was also found that the rate of testing used in this testing program had no effect on the fatigue properties of lighweight aggregate concrete. Comparison of the data collected in this study with data collected in a previous study indicated that the fatigue properties of lightweight aggregate concrete are not significantly different than the fatigue properties of normal weight concrete.

INTRODUCTION

Lightweight aggregate concrete is rapidly becoming a very useful building material. In many instances the additional cost of the lightweight aggregate is more than justified by the savings in total cost of the entire structure. The upper deck of the San Francisco-Oakland Bay Bridge was paved with concrete weighing approximately 104 pcf. It is estimated that the savings made possible by the use of lightweight concrete amounted to about $3,000,000 (1)*. Close examination one year after completion showed that the lightweight concrete was equivalent to heavy concrete in every respect.

One of the first applications of lightweight concrete was its use in the hulls of ships. The successful use of lightweight concrete in the construction of ships brought interest in the material as an economical substance for building construction. Several studies have shown that a substantial savings can be accrued for most types of concrete buildings if lightweight concrete is used (2, 3).

Heat insulation is an important attribute of lightweight concrete which makes it even more desirable as a building material. It has been found that concretes can be made that weigh between one-third and two-thirds as much as normal concrete and have thermal conductivities of about one-half to one-fourth that of regular sand-and-gravel concrete (4).

Although lightweight aggregate concrete has very desirable qualities for structural use, comparatively little is known about the material. Many tests have been carried out to determine the properties

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* Numbers in parentheses pertain to references listed at the end of this thesis.

of the various aggregates available, but relatively few tests have been made on the properties of the finished concrete. One property that is of vital interest in every structural material is its resistance to repeated loading.

Many tests have been conducted to determine the resistance of some materials to repeated loading. It has been found for example, that for most steels a definite endurance limit can be found. Below this endurance limit the material can apparently withstand an infinite number of stress applications without failing. A graph of the percentage of ultimate stress versus the log of the number of stress applications has a negative slope and is referred to as the S-N curve. When this curve is plotted for a ferrous metal, the curve levels off and becomes horizontal at the endurance limit. If no endurance limit can be established, the curve continues to have a negative slope. Concrete is a material in which no leveling off point has been established (5).

Lightweight aggregate concrete has proven itself to be a useful structural material and its uses and applications are becoming more numerous as engineers learn more about it. Since it proved to be a cost-saving material in the San Francisco-Oakland Bay Bridge and several other major bridges, there is reason to believe that it will continue to be used in these applications. Its properties of light weight and heat insulation make it very desirable as a building material. In bridges and elsewhere, lightweight aggregate concrete when used as a material must be able to withstand repetition of loading. Hence, more information is needed on the fatigue resistance of this material. The work reported in this thesis had as its major objective, the establishment and comparison of the S-N relationship of two lightweight aggregate concretes.

LITERATURE REVIEW

Before the testing of this study was begun an extensive survey of literature was conducted on the subjects of lightweight aggregate and fatigue of concrete. Literature on the characteristics of fatigue failure and the fatigue of metals was also reviewed. Most of the literature discussing the fatigue properties of metal also describes the phenomenon and theory behind the failure of materials subjected to repeated loading. Hence the subsequent discussion of mechanics of fatigue will be based primarily upon the results of fatigue tests conducted on metals. Less is known about the mechanics of failure in concrete but the following discussion will suffice for a background in fatigue failures.

Nomenclature

Before presenting the basic mechanics of fatigue failure it was felt that the nomenclature used in this thesis should be defined. In 1949 the American Society for Testing Materials published a standard set of symbols and definitions for use in fatigue testing (6). Before this time there was no standard nomenclature, and terms presented in the early literature were interchanged and confusing. The definitions shown below are taken from the Manual on Fatigue Testing (6).

Stress Cycle - A stress cycle is the smallest section of the stress time function which is repeated periodically as shown in Figure 1.

Nominal Stress, S - The stress calculated on the net section by

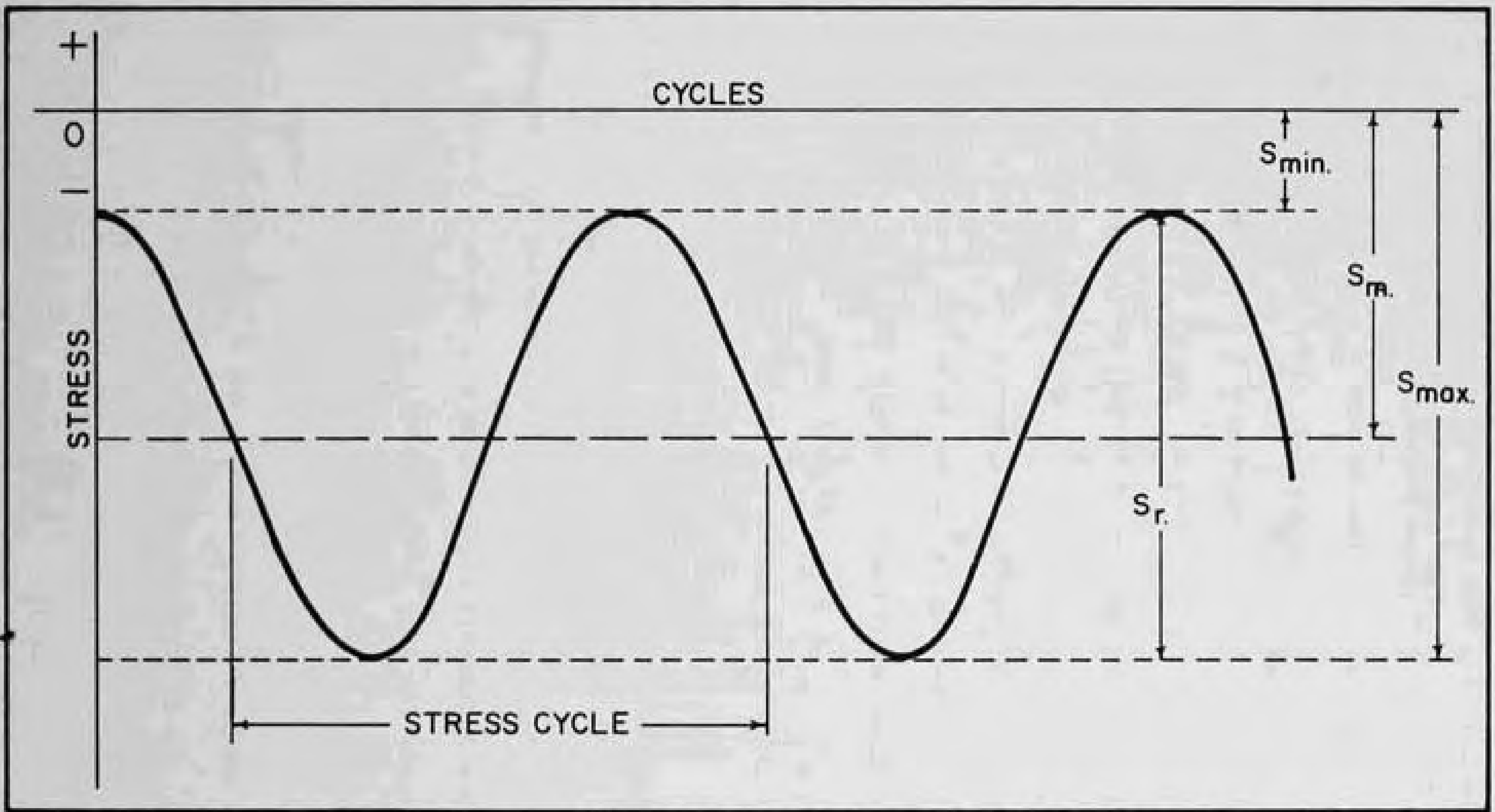

FIGURE I. TYPICAL FLUCTUATING COMPRESSIVE STRESS ENCOUNTERED IN FATIGUE TESTING simple theory such as P/A without taking into account the variation in stress conditions caused by geometric discontinuities such as holes, grooves, fillets, and etc.

Maximum Stress, S_{max} - The highest algebraic value of the stress in the stress cycle, tensile stress being considered positive and compressive stress negative.

Minimum Stress, S_{min} - The lowest value of the stress in the stress cycle, tensile stress being considered positive and compressive stress negative.

Stress Range, S_r - The algebraic difference between the maximum and minimum stress in one cycle, that is $S_r = S_{max} - S_{min}$.

Mean Stress, S_m - The algebraic mean of the maximum and minimum stress in one cycle, that is $S_m = (S_{max} + S_{min})/2$.

Fatigue Life, N - The number of stress cycles which can be sustained for a given test condition.

S-N Diagram, - A plot of the stress versus the number of cycles to failure.

Fatigue Limit (or Endurance Limit), S_e - The limiting value of stress below which a material can presumably endure an infinite number of stress cycles, that is, the stress at which the S-N diagram becomes horizontal and appears to remain so.

Fatigue Strength, S_n - The greatest stress which can be sustained for a given number of cycles without fracture.

Mechanism of Fatigue Failures

Before discussing the mechanism of fatigue failure, it might be worthwhile to review the method in which a metal fails when subject to loads greater than its ultimate tensile stress. Most metals are made up of a crystalline structure which is not destroyed by the deformations arising from tensile stress. If two adjacent crystals are examined as the tensile stress is increased, it will be found that the bond between the two crystals is not disturbed, but that the yielding will take place along slip planes at a limited number of places within each crystal. Thus if the surface of the two crystals were polished before elongation, they would look like steps after elongation. As the stress is increased the number of slip planes increases until a condition of failure occurs by tearing of the crystals along the slip planes (7).

There has been much controversy about what actually happens when slip occurs within a crystal. The general feeling is that the molecules along the slip planes are arranged in such a way that the inter-atomic bonds are destroyed between the two faces of slip. As slip takes place, a thin film of amorphous metal is produced which cements the two faces together. This cementing creates a stronger bond than previously existed and hence the metal has a greater resistance to slip. This greater resistance to slip is called strain hardening (7, 8).

Fatigue failure involves three stages (9):

1. Slip occurs resulting in strain hardening and lattice distortion.
2. The fatigue crack starts.
3. The crack spreads along the path of least resistance, due to stress concentration. This proceeds until the

cross-section of the metal is reduced so much that the remaining metal tears or breaks away suddenly.

It has been observed by many investigators that a load deformation curve forms a hysteresis loop when a beam is first loaded and then unloaded. This separation of the loading and unloading curves is due to energy absorbed by the metal and is observed even though the metal returns to its original shape. The theory has been advanced that if the hysteresis loop remains at a constant width throughout fatigue testing, regardless of that width, the specimen will not fail. If the width of the hysteresis loop does increase, however, the specimen will eventually fail (7).

It is believed that fatigue failures start at points of inhomogeneity within a metal. Most materials in which fatigue would be of concern cannot be made perfectly homogeneous so that they cannot be subdivided indefinitely without changing the properties. Hence the stresses that are computed for a specimen are an average of the total stress across any one section. There is likely to be points throughout a specimen at which the stress is considerably larger than the computed average. The steel used in machines and structural parts today is quite ductile and can adjust to these stress concentrations if the load is steady. If the load fluctuates, however, a crack is likely to form at the point of stress concentration and the failure due to fatigue progresses from this crack (7).

Very little is known about the nature of fatigue cracks during their initial stages. Attempts have been made to observe the cracks in their early stages by microscope, but these have met with little success. There have been several theories suggested as to the cause of cracking under the action of repeated loading. Some believe that cracks are started at the surface of the metal and are caused by high stress concentrations at existing irregularities (7). It has been reported that the highest real stress concentration is less than ten times and probably about nine times as great as the average stress. Theoretically, a sharp crack will produce an infinite stress concentration (10). The stress concentration around a circular hole is about three times the average stress (11).

Regardless of where the cracks start, once they have started, they progress along the path of least resistance. Ultimate failure may or may not result, depending on the magnitude of the pulsating load. As the load is applied, three things may happen:

1. The size of the crack is increased.
2. The curvature at the end of the crack becomes less sharp and thus tends to relieve the stress concentration.
3. Slip and fracture occur at the end of the crack which tend to relieve stress concentration and strengthen the metal by strain hardening.

If the applied load does not exceed a certain limiting value, equilibrium will be established in which the reduction of cross-sectional area will be counterbalanced by a reduction in concentrated stress and strain hardening. If the applied load exceeds this limiting value the cracking proceeds until failure.

Fatigue of Metals

There have been several literature reviews of the fatigue of metals. The results of two of these reviews are discussed in References (7) and (8). It can be noticed from these reviews that investigators realized in the early part of the 19th century that a metal would fail at a stress below the static ultimate stress if that stress were reapplied repeatedly. It is felt that another review of the literature on the fatigue of metals would yield no further information, but that a summarization of the findings of the early investigators would be helpful to the understanding of the basic concepts of fatigue. Such a summarization follows:

1. A definite fatigue limit can be established for ferrous metals and ferrous alloys. At this fatigue limit an elbow is apparent in the S-N diagram.
2. Most non-ferrous materials do not have a definite fatigue limit. There is no elbow in the S-N diagram and the curve continues to slope downward.
3. Flaws on the surface and discontinuities in the interior of a material have a detrimental effect upon the fatigue strength of the material.
4. Speed of testing and temperature have no effect upon the fatigue strength unless the temperature is high. If the stress is applied at such a rate that the heat gain due to hysteresis cannot be dissipated, the rate of testing could effect the fatigue strength.
5. If a metal is repeatedly stressed below its fatigue limit, strain hardening will occur and the fatigue life of the material will be increased.
6. As the mean stress increases, the range of stress which will endure a given number of cycles decreases.
7. If fatigue testing is stopped and the specimen allowed to rest

for a while, partial recovery will take place and the fatigue life will be increased.

The fatigue strength of metals is best correlated with the ultimate tensile strength. Several investigators have attempted to establish relationships between fatigue strength and other properties of steel but these have not led to very satisfactory results (8).

Fatigue of Concrete

In recent years several investigators have completed several extensive literature reviews on the fatigue of concrete. In 1927 Moore and Kommers published their text on The Fatigue of Metals which contained a chapter on the fatigue of cement and concrete (7). This chapter presented the tests and results of many investigators before that time. In 1958 Nordby published a paper summarizing the most important literature on the fatigue of concrete (5). In his literature search, Nordby reviewed more than 100 publications including many in foreign literature.

The work reported in this thesis is part of a series of fatigue testing programs to be conducted at Purdue University. Previous to this study one was conducted by Antrim at Purdue University to determine the effect of entrained air on the fatigue properties of concrete (12, 13). Antrim included an extensive literature review in the write-up of his work (13). Since the available literature has been reviewed and summarized at least twice in the past few years, it is felt that the inclusion of another complete summary in this thesis would not be warranted. Instead, the conclusions drawn from the previous reviews will be presented along with a review of the literature which has been published since these reviews were completed.

The conclusions drawn by Antrim (13) in his review of literature are quoted below:

1. The early investigations indicate that there is a definite stress that is approximately 50 to 55 per cent of the ultimate static stress below which concrete can undergo fatiguing action indefinitely, and above which the number of cycles to failure decreases as the stress increases.

2. There appears to be little variation in this limit that is due to type of testing (compression, flexure, and tension) and speed of testing.

3. Permanent set occurs during the earlier stages of the fatigue action. If the maximum stress is below the fatigue limit, the permanent set reaches and maintains a constant value. Stresses above the fatigue limit cause progressive deformation.

4. When the applied stress is below the fatigue limit, the modulus of elasticity reaches and maintains a constant value.

5. The repetition of a stress which is below the fatigue limit appears to increase the strength of concrete.

Nordby (5) goes further in summarizing the results of these same investigations and his conclusions are quoted below:

1. Under repetitive load the modulus of elasticity changes in various ways depending upon the intensity of load. The secant modulus decreases with repeated load; the slope of the stress-strain curve may decrease in the lower part of the curve and increase slightly in the upper portion to become concave upward.

2. Age and curing has a decisive effect on the fatigue strength. Inadequately aged and cured concrete is less resistant to fatigue than well-aged and cured concrete.

3. Rest periods seem to increase the endurance of concrete although test results are very scant.

4. Fatigue strength decreases slightly with leaner mixes and higher water cement ratios (data not extensive).

5. As the range of stress is decreased the upper limit of the stress (fatigue strength) is increased substantially. This phenomenon can be represented by the Modified Goodman Diagram.

A summary of the results of the literature on the fatigue of reinforced concrete, taken from Nordby's paper is as follows:

1. Most failures of reinforced beams were due to failure of the reinforcing steel. The failures seemed to be connected with severe cracking and the possible stress concentration and/or abrasion connected these cracks. Beams critical in longitudinal reinforcement seemed to have an endurance limit of 60 to 70 per cent of the static ultimate strengths for one million cycles.

2. Often times it was pointed out that the concrete in the compression zone behaves much the same way as axially loaded compression specimens. There is certainly no assurance of this, since it is not true for static tests and of course there is a strain gradient in the beam which does not exist in compression specimens. No fatigue compression failures were noted in any of the beams reported except those of LeCamus.

3. On occasion beams failed in diagonal tension fatigue but the real cause of failure was obscured by bond and shear combination failures. Tests have been reported in which beams have failed in shear in repeated loads as low as 50 per cent of the ultimate strength. Data are very scarce on this phase.

4. Beams accumulate residual deflections under extensive fatigue loading much the same as plain concrete specimens; but recover somewhat during rest periods.

In addition to the conclusions quoted above on fatigue properties of plain and reinforced concrete, Nordby also drew conclusions on the fatigue properties of prestressed concrete which are as follows:

1. In none of the tests did concrete fail by fatigue. The current working stresses seem to give adequate protection in this regard.

2. Fatigue failure of stressing wires or strands was the cause of all failures reported. These failures seem to be related to the extent and severity of the cracks.

3. Bond failures were rare and found only under unusual circumstances, i.e., short beams, short shear span.

4. The ultimate strength of prestressed beams for static loads was unaffected by repetitive loading if they did not fail by fatigue.

5. Safety factors seemed to be approximately "two" against fatigue failure for most of the beams tested.

6. Prestressed beams seemed to be superior to conventional beams for resisting fatigue loading. In fact, in a recent paper, Eckberg and Walther analytically verified this by relating the modified Goodman diagram of both the concrete and prestressing steel to the theoretical stresses in both types of beams.

Several papers have been written on the fatigue properties of concrete since these extensive literature reviews were completed. Antrim and McLaughlin found that intentionally entrained air in a concrete did not affect the fatigue properties of the concrete (12). It was apparent from their tests, however, that more consistent fatigue results could be obtained with air-entrained concrete than with non-air-entrained concrete. These findings were the results of fatigue tests on 65 plain concrete cylinders 3 inches in diameter and 6 inches in length.

In 1958 McCall attempted to show a correlation between the S-N curve and the probability of failure (14). McCall conducted fatigue tests on 3 x 3 x 14-1/2 inch plain air-entrained concrete beams. The beams were tested until they had failed or endured 20 million repetitions of load. He concluded that the S-N curve for concrete did not level off in the neighborhood of 20 million cycles and that the probability of fatigue failure at 20 million cycles is slightly less than 1/2. A graphical relationship between the S-N curve and the probability of failure is shown.

Assimacopoulos, Warner, and Eckberg have reported results of tests on 34 plain concrete cylinders (15). Twenty-five 2 inch by 4 inch cylinders were tested in direct compression at a speed of 9,000 cycles per minute. Nine other cylinders of the same size were tested at a rate of 500 cycles per minute. Specimens were tested at different minimum and maximum stress levels at each speed and the temperature changes of the specimens were recorded as the tests progressed. No appreciable temperature change was measured in the cylinders tested at 500 cycles per minute but an increase in temperature of nearly 100°F was measured in the specimens tested at 9,000 cycles per minute. The temperature increase was generally in direct proportion to the range of stress variation but no adverse effect of this temperature increase could be seen in the test results. The authors concluded that, although the amount of data was quite small, no difference in the fatigue properties could be detected for the rates of loading used.

Several recent tests have been made to determine the fatigue action of reinforced concrete beams. Chang and Kesler tested 25 beams having a 4 x 6 inch cross-section and containing 1.02 per cent reinforcing steel (16). The beams were loaded at the third points of a 60-inch span at a rate of 440 cycles per minute. The beams of this study displayed four different modes of failure involving reinforcement, diagonal cracking, and shear-compression.

Stelson and Cernica tested eleven over-reinforced beams (17). They determined that the fatigue limit of these beams was between 60 and 65 per cent of the static ultimate load. The beams failed in diagonal tension.

PURPOSE AND SCOPE

The purpose of this study was to determine the fatigue characteristics of lightweight aggregate concrete and, further, to investigate the effects of varying the strength and aggregate proportions on this property. Finally, it was the purpóse of this study to determine the fatigue limit of lightweight aggregate concrete if the data indicated that one existed.

Specimens were made from concrete of two different proportions. One was designed for a high strength and relatively high percentage of fine material. The other mix was designed for a lower strength and was designed for a high percentage of coarse material. Both mixes were designed for the same slump and same percentage of total air. Batches were made periodically so that specimens were tested at nearly the same age. Specimens were tested from each batch at stress levels of 40, 50, 60, 70, and 80 per cent of the static ultimate compressive strength of the respective batch.

TESTING PROGRAM

The testing program was divided into two parts. In the first part tests were conducted on concrete designed to have a static compressive strength of 3,500 psi. This concrete was called the low-strength concrete and will be referred to as the LL series. The second part consisted of conducting tests on concrete designed to have a static compressive strength of 6,000 psi. This concrete was called the high-strength concrete and will be referred to as the HL series. Each series was made up of five batches of concrete and from each batch 30 cylinders 3 inches in diameter and 6 inches in length were cast.

All of the cylinders from any batch were cured for 28 days, after which, about one-half of them were placed in an oven at 105°C for about four days to prevent further hydration. The remaining cylinders were placed in storage at room conditions so that they could be tested at a future date. The object of the future testing is to determine whether or not drying of the specimen in an oven will affect the fatigue properties.

After the drying was completed the specimens were removed from the oven and capped. Five specimens were then chosen at random and tested in static compression so that an estimate of the batch strength could be made. Fatigue tests were conducted on about five of the remaining cylinders at various stress levels. If any specimens from a batch remained after the fatigue testing was completed, they were tested in static compression to determine whether any change in the batch strength occurred during the fatigue testing.

Tests were also conducted on one batch from the HL series to determine if the rate of load application had any effect on the fatigue properties of lightweight concrete.

Materials

The aggregate used in this study was an expanded shale produced in a rotary kiln. It was shipped from the producer in central Indiana to Purdue University in bags containing aggregate of two sizes. The gradations of these two aggregate sizes are shown in Tables 1 and 2.

Since the physical properties of expanded shale, such as specific gravity and absorption characteristics, are extremely variable, the use of these properties in the design of the concrete was not practical and these properties were not determined.

Type I portland cement manufactured in central Indiana was used in both mix designs. All of the cement used in this study came from one clinker batch (laboratory designation 315) and it was assumed that its characteristics did not vary significantly.

Darex was added to the mixing water of all batches as an air-entraining agent.

Mix Design

The mixes were designed in accordance with the ACI "Proposed Recommended Practice for Selecting Proportions for Structural Lightweight Concrete," (18) except that the specific gravity factor was not computed.

The combination of fine and coarse aggregate that would yield the maximum density (minimum voids) was determined by plotting the unit

Table 1

GRADATION OF COARSE AGGREGATE

Sieve Size	Per Cent Passing
3/8 inch	100.0
No. 4	30.1
No. 8	2.9
No. 16	1.9
No. 30	1.8
No. 50	1.7
No. 100	1.6
No. 200	1.4

Table 2

GRADATION OF FINE AGGREGATE

Sieve Size	Per Cent Passing
No. 4	100.0
No. 8	93.3
No. 16	71.7
No. 30	48.2
No. 50	27.1
No. 100	13.2
No. 200	7.2

weight against the percentage of fines. This plot is shown in Figure 2. It was decided that 60 per cent of the aggregate should consist of the finer fraction in the LL series and 65 per cent of the aggregate should consist of the finer fraction in the HL series.

Trial mixes were used to determine the required amount of water and the proper cement factor. The amount of Darex needed to produce the design air content was also determined from the trial mixes.

The low-strength concrete was designed for a strength of 3,500 psi, a slump of three inches, and an air content of seven per cent. A cement factor of 5.8 bags per cubic yard was required to produce the design strength.

The high-strength concrete was designed for a strength of 6,000 psi, a slump of three inches, and a total air content of seven per cent. A cement factor of 9.6 bags per cubic yard was required to produce the design strength.

Mixing Procedure

Both the fine and coarse aggregates were stored in covered barrels in the laboratory. The aggregate was sieved so that it could be stored in four main size groups for better control of gradation. The coarse aggregate was stored in two barrels, one containing material retained on the No. 4 sieve and the other containing material passing the No. 4 sieve and retained on the No. 8 sieve. The fine aggregate was stored in two barrels, one containing material retained on the No. 16 sieve and the other containing material passing the No. 16 sieve.

It has been found by several investigators that the moisture content of lightweight aggregate immediately prior to mixing has little

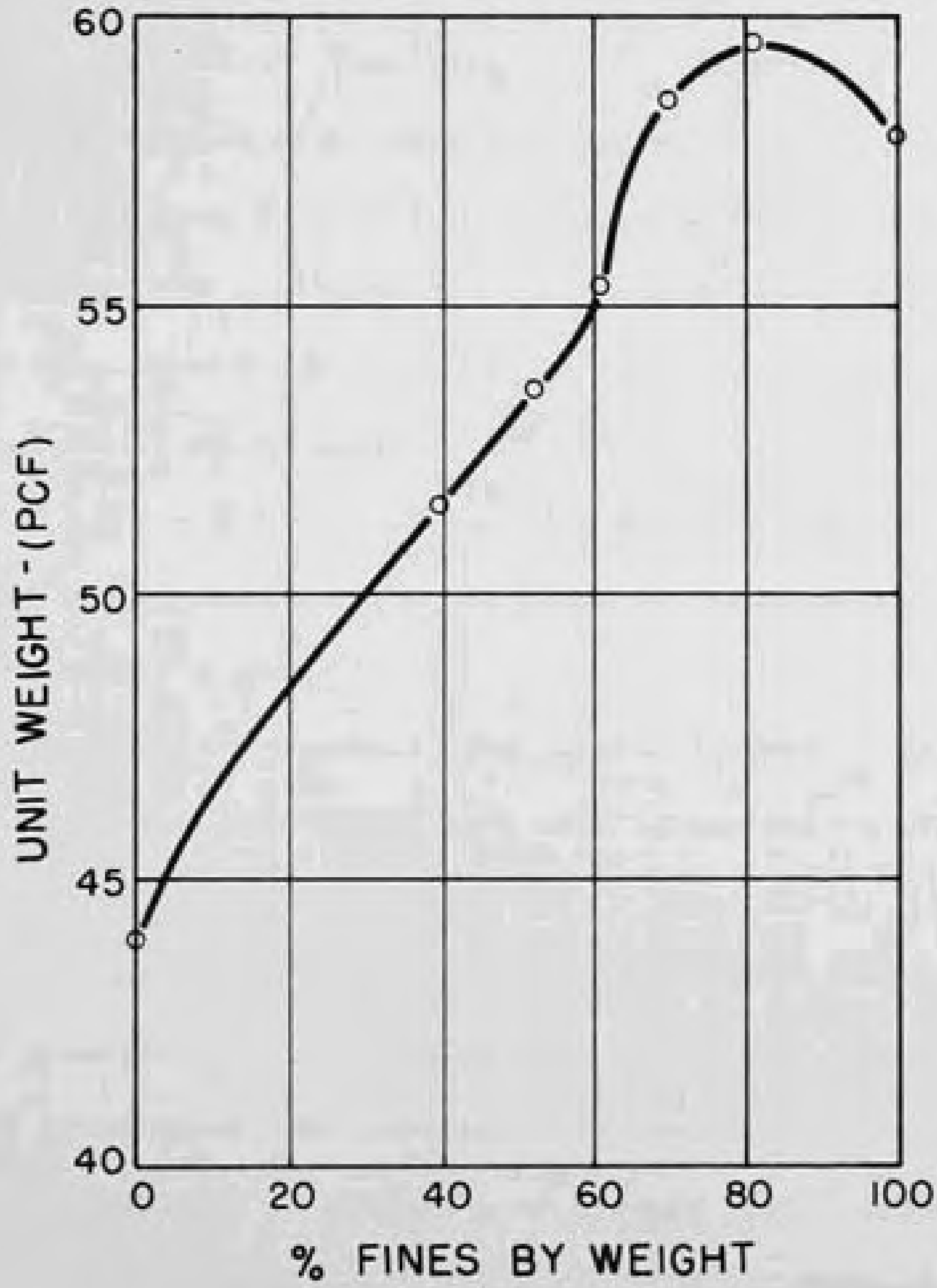

FIGURE 2. MAXIMUM DENSITY CURVE

effect on the compressive strength of the concrete (18). For this reason no effort was made to saturate the aggregate or determine its water content before mixing.

The mixing was done in a 1-1/2 cubic foot capacity, Lancaster, tub type, counter current mixer. Two-thirds of the mixing water was placed in the mixer along with the fine and coarse aggregate and allowed to mix for five minutes. It was felt that this mixing period would satisfy the initial absorption of the aggregate and thus produce more uniform results. The cement was then added and allowed to mix for one minute with the moistened aggregate. Finally the remainder of the mixing water and the Darex were added and the batch was mixed for an additional two minutes. The mixing water was adjusted during the final two minutes so that the slump would be about 3 inches.

Three measurements were made on the plastic concrete. Slump, unit weight, and air content measurements were made in accordance with ASTM Designations: C 143-58, C 138-44, and C 173-58 respectively. The concrete used for these tests was discarded so that no chance of contaminating the material left in the mixer would result. In the case of the unit weight measurement, the one cubic-foot measure was replaced by a measure having a capacity of one-tenth of a cubic foot because the size of the batch was small. Modifications of the air content measurements are described in the following section.

Air Determination

The air content of the low-strength concrete was determined by the gravimetric method as described in ASTM Designation: C 173-58. When this method is used a given amount of plastic concrete is placed in a device called a Rollameter. The Rollameter is then filled with water and rolled until the air in the concrete is displaced by the added water. The laboratory was not equipped with a Rollameter at the time this study was in progress but a PCA type meter was available which was similar to the Rollameter in every way except size.

A borrowed Rollameter was used to determine the air content of the LL series. Air content was also determined for the LL series by rolling the PCA type meter in a fashion similar to that used for the Rollameter.

The PCA type meter was calibrated with the Rollameter from air content determinations made on the LL series and several trial mixes. Air content determinations on the high-strength concrete were made with the calibrated PCA type meter.

Molding and Curing of Specimens

Thirty specimens, 3 inches in diameter and 6 inches in length, were cast from each batch of concrete in accordance with ASTM Designation: C 192-57. Since small molds were used, it was necessary to replace the 5/8-inch diameter rod by a 3/8-inch diameter rod.

Immediately after casting, the specimens were covered with metal pans and moistened rags to prevent evaporation while they were stored at room temperature in the molds for 24 hours. The specimens were then removed from their molds and stored for 27 days in a saturated lime solution at a temperature of about 70°F.

Drying

At an age of 28 days, all of the specimens were removed from the saturated lime solution. Approximately one-half of the cylinders from each batch were placed in an electric oven and allowed to dry at 105°C until they reached constant weight. The remaining specimens from each batch were placed on shelves in the laboratory to dry at room temperature. Oven drying usually required four days.

Capping

After the specimens were removed from the drying oven, they were allowed to cool for 24 hours before capping. Caps were then placed on each end of all cylinders in the device shown in Figure 3. A sulfur-carbon compound (trade named Vitrobond) was used for the capping material. The capping material was heated to approximately 275°C before being poured into the molds. The caps had a thickness that varied from 1/16 of an inch to 3/16 of an inch, depending upon the end conditions of the specimens.

An attempt was made to arrange the testing program so that fatigue tests could be started within four or five days after oven drying. Several power failures and machine break-downs interrupted the testing program, however, so that a time lag of from two days to two weeks occurred between capping and fatigue testing. It is assumed that this time lag had no effect on the fatigue properties of the concrete.

Static Compression Tests

Static compression tests were conducted on five randomly chosen specimens from each batch to estimate the ultimate strength of the batch. These tests were performed in a Riehle screw-type testing machine with a Graham variable speed drive. The machine had a capacity of 50,000 lbs. and the no-load head speed was set at 0.05 inches per minute as prescribed in ASTM Designation: C 39-56T.

FIGURE 3. CAPPING DEVICE

It was originally planned to test the static compression specimens one day after capping. After several power failures delayed the fatigue testing program, however, it was decided that the static compression tests should be delayed until fatigue testing was about to begin. Thus the age at which static tests on the specimens were conducted varied considerably as shown in Tables 8 through 17 of Appendix A.

Fatigue Tests

Eight specimens were chosen at random from each batch to be tested under a pulsating load. These tests were conducted at 40, 50, 60, 70, and 80 per cent of the estimated static compressive strength of the batch. Specimens were tested in direct compression only and a minimum stress of between 70 and 170 psi was maintained on all specimens to eliminate any possibility of impact. The maintaining of a constant or near constant minimum load required the range of stress to vary between stress levels. This variable was not considered, however.

In some cases, where power failures or testing machine break-downs were frequent, it was not possible to test specimens from each batch at all stress levels. In the HL series, no specimens were tested at the 40 per cent stress level after the first two batches had been tested and it was found that at 40 and 50 per cent of the static ultimate strength specimens endured more than ten million cycles of load. On this basis it was felt that further testing of specimens at the 40 per cent stress level would yield no additional information. Ten million cycles was selected at the beginning of the study as the maximum number of cycles any specimen would be permitted to endure. The data at the 40, 50, and 60 per cent stress levels are not complete (fewer than five specimens were tested to failure) because some or all of the specimens endured ten million cycles.

Other causes of incomplete data were power failures and machine break-downs which ruined all of the remaining specimens in a batch. Once the testing of a specimen had stopped, the specimen was discarded and a new specimen was tested. No attempt was made to test at the 90 per cent stress level because it was found in a previous investigation that, with the equipment available, the specimen would fail before the load could be established (13).

The Krouse-Purdue Machine

At the beginning of this study only the Krouse-Purdue fatigue testing machine was available. This machine is of the constant deflection type which derives its force from hydraulic pressures acting on a piston. The Krouse-Purdue machine is shown in Figure 4.

Two components of load are applied by the Krouse-Purdue Machine. The first is an average preload which is proportional to the difference in average pressures existing at opposite ends of a hydraulic cylinder. This load is automatically controlled by a hydraulic make-up pump. The pulsating load is controlled by varying the throw of an eccentric crank. This causes a loading which is alternately larger and smaller than the preload. Both loads can be adjusted while the machine is in operation. A simplified line diagram of the Krouse-Purdue machine is shown in Figure 5.

The specimen is held in place by an adjustable load screw which extends through the upper head of the machine. A reversible motor is used to adjust the load screw and provide the required testing space. When the required testing space has been established, a locking nut is

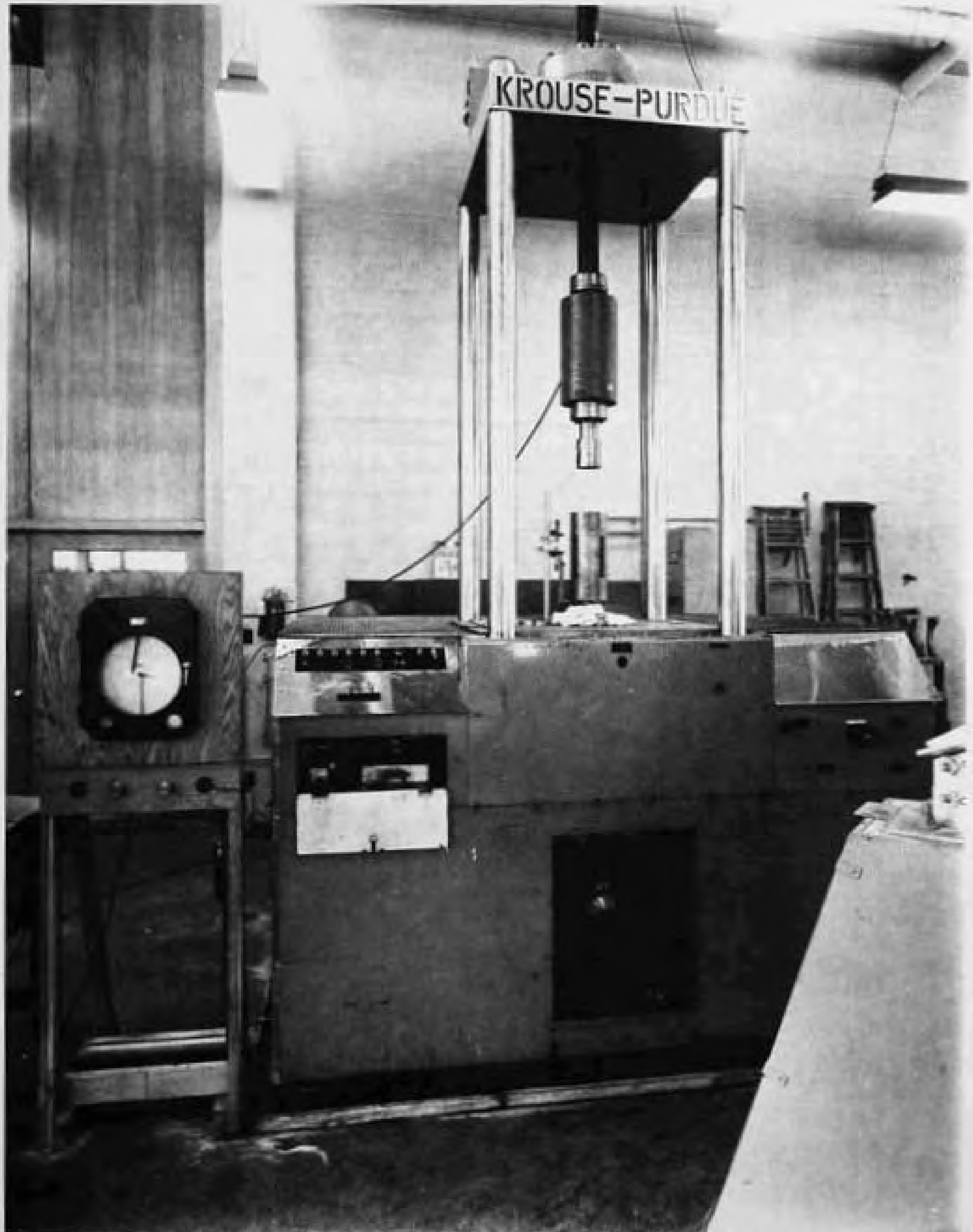

FIGURE 4. KROUSE-PURDUE FATIGUE MACHINE

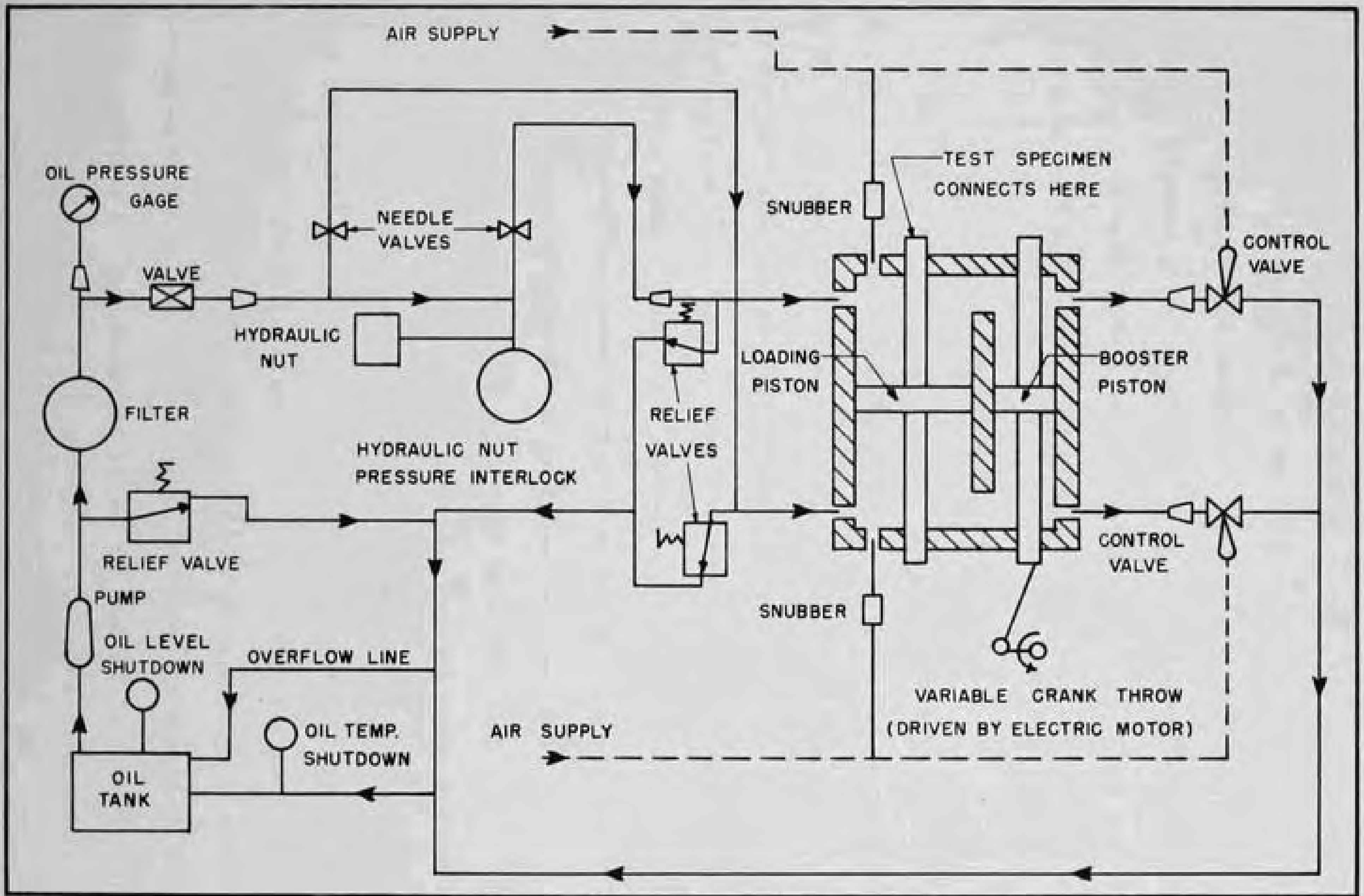

FIGURE 5. HYDRAULIC SYSTEM OF FATIGUE MACHINE

closed on the screw by means of a hydraulic cylinder. A close-up of the specimen holder and automatic shut-down device is shown in Figure 6.

The Krouse-Purdue machine has a capacity of ± 60,000 lbs. and operates at 1,000 cycles per minute. Loads are measured by an electronic system which is actuated by a Baldwin-Lima-Hamilton type U-1, SR-4 load cell. The load cell is an integral part of the loading screw. Loads can be measured to within ± 100 lbs.

The Amsler Machine

At about the mid-point of the fatigue testing program an Amsler fatigue testing machine was installed in the materials testing laboratory at Purdue University. This machine is based on the same principle as the Krouse-Purdue machine except that the loading jacks are separated from the pulsator. Tubing connects the pulsator and the loading jack.

The preload of the Amsler machine is the minimum load to be applied and the pulsator increases the load from this minimum to the desired maximum. Loads are transmitted by hydraulic pressure through the tubing to the load jack which is mounted on a specially built frame. The jack develops its force by pushing against the frame and the concrete specimen which is placed on a bearing plate on the floor. The pulsator is shown in Figure 7 and the loading jack is shown in Figure 8.

This machine has a capacity of 110,000 lbs. and can operate at speeds of either 250 cycles per minute or 500 cycles per minute. Loads are measured by two gauges mounted in the hydraulic system of the machine. It is possible to read these gauges to the nearest 100 lbs.

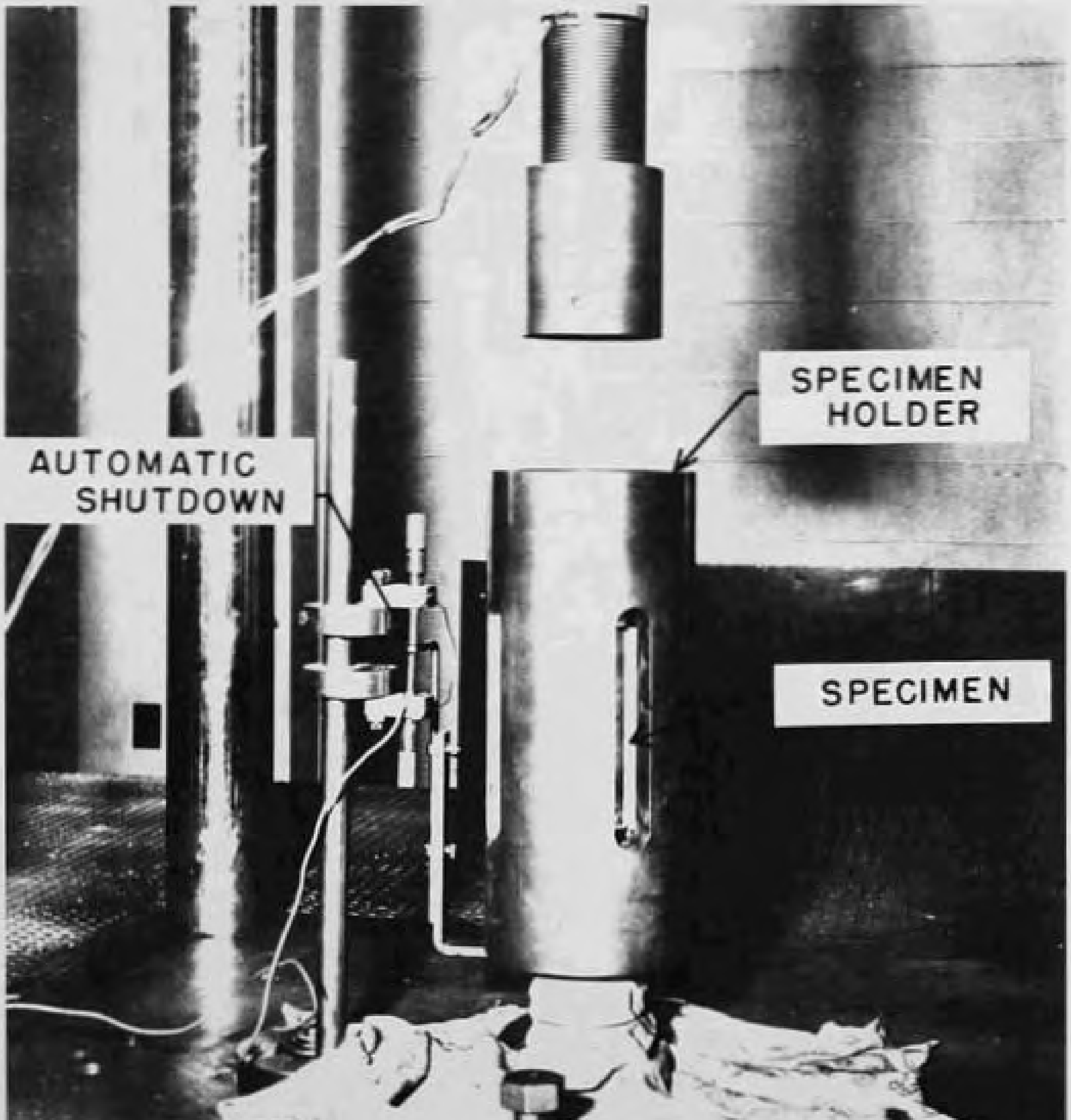

FIGURE 6. SPECIMEN HOLDER

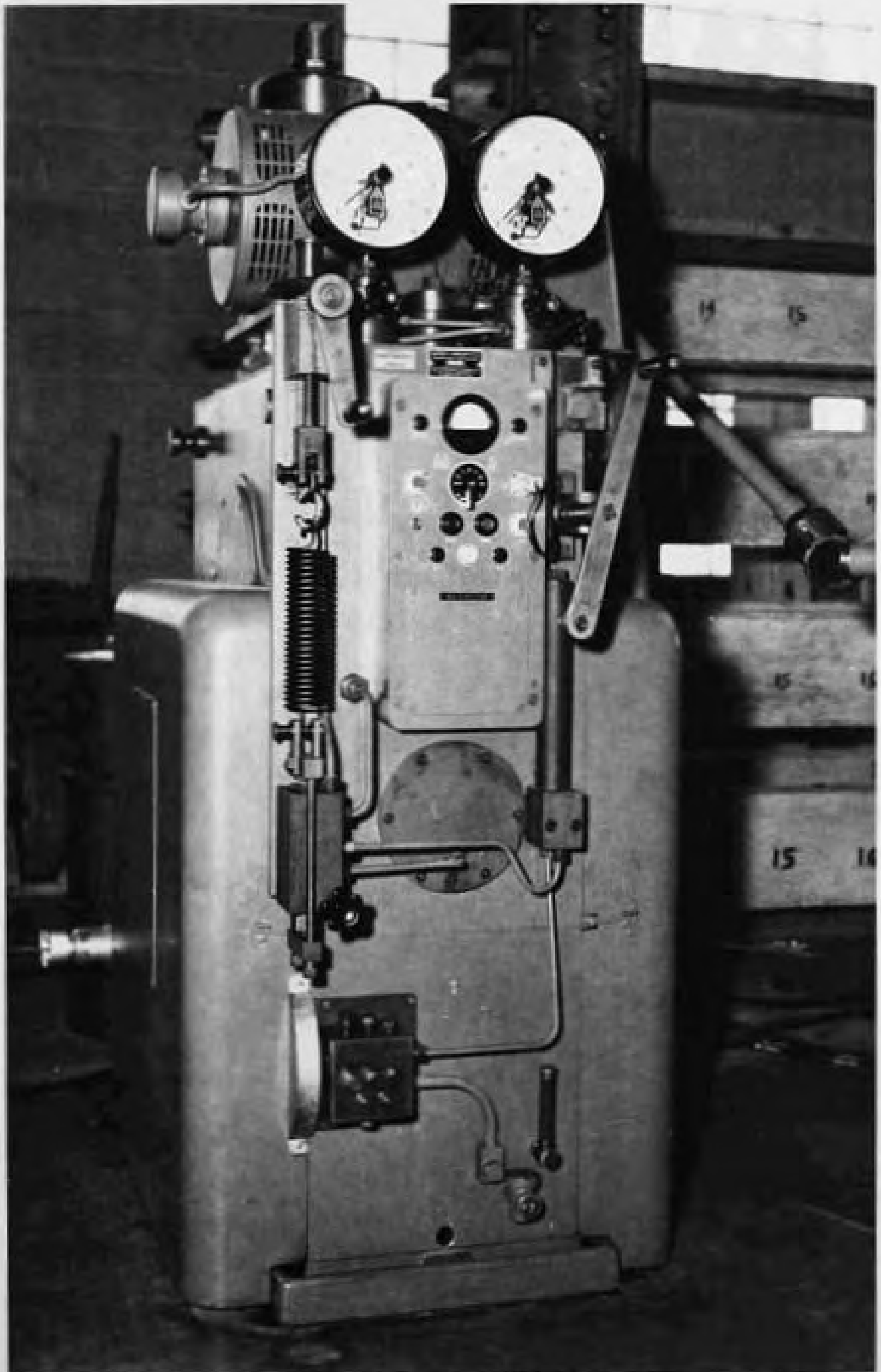

FIGURE 7. PULSATOR

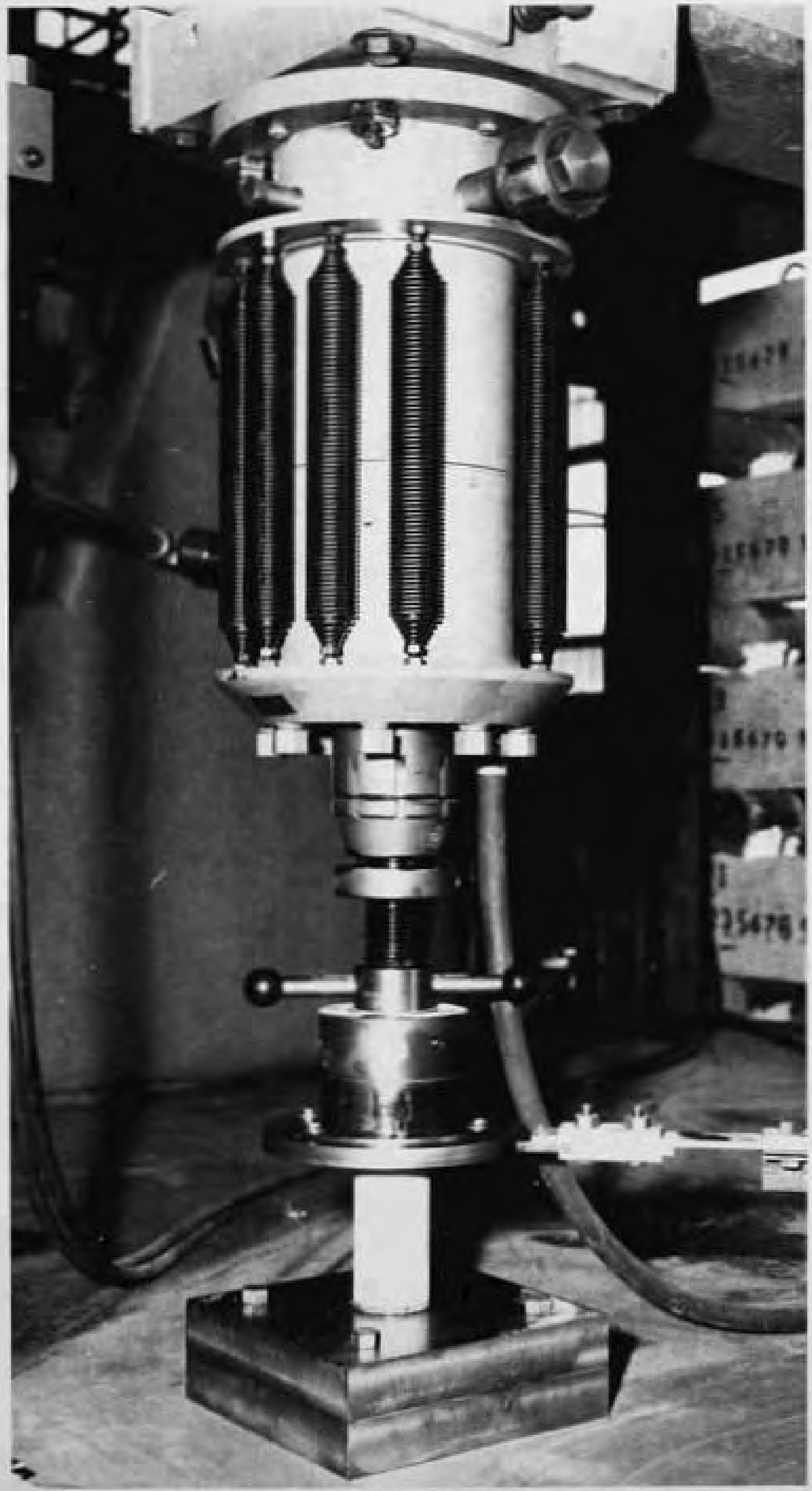

FIGURE 8. LOADING JACK

Fatigue Testing at Different Speeds

Since the Krouse-Purdue machine tested at a rate of 1,000 cycles per minute and the Amsler machine was used at the maximum speed of 500 cycles per minute, it was felt that a test should be carried out to see if this variation would affect the fatigue test results. Batch HL 1 was selected for this purpose and it was decided to test as many cylinders as possible in each machine. All of the specimens from batch HL 1 were, therefore, oven-dried. Fatigue tests were conducted on these specimens, at the 80 per cent stress level only, in the same way as they were in all other batches except that one-half of the specimens were tested in each machine.

Static Compression Tests to Determine Increase in Strength

A period of at least two weeks was required to complete the fatigue testing on any one batch. It, therefore, seemed reasonable to determine the increase in strength which took place over this period. After fatigue testing had been completed the remaining specimens in the batch were tested in direct compression if enough specimens remained to make a reasonable estimate of the batch strength. The procedure used was identical to that used when the first estimate of batch strength was made.

DISCUSSION OF RESULTS

The discussion of the results of this study has been divided into three parts. The first part is concerned with physical properties of the plastic concrete and the static tests on the hardened concrete. The discussion of the fatigue testing data is presented in the second part and the third part is concerned with the comparison of the results of this study with the results of a previous study.

Analysis of Mix Data

Since each series consisted of a group of five separately mixed batches, it was necessary to estimate the properties of each series from the properties of the individual batches. The purpose of this study was to compare the fatigue properties of the two classes of concrete but before this could be done, it was necessary to show that other properties of the two concretes were different and that each batch within a series was not different than the other batches within the series.

A statistical procedure known as the analysis of variance was used to test for any significant difference between the average batch strengths (19). When using this procedure, two estimates are made of the total variance in individual specimen strengths, these being the sum of the individual variance within each batch and the variance between batches. It is known that the ratio of two variances follows an F-distribution. If the calculated ratio of these two variances is less than the theoretical F-value at some significance level there is no statistically significant difference between the individual batch averages and the apparent difference can be attributed to chance. If the calculated F-value is larger than the theoretical F-value at some significance level then there is a statistically significant difference between the batch means.

The analysis of variance is valid only if the individual values follow a normal distribution and if the batch variances are equal. It is assumed in this analysis that the individual strength measurements are normally distributed. When this assumption is not satisfied the consequences are not great (20). Bartlett's test was used to test for a significant difference in batch variances (19). This test assumes that the value of M/C computed in Tables 18 and 20 of Appendix B follows a chi-squared distribution.

Low-Strength Concrete

The data for the low-strength concrete are shown in Tables 8 through 12 in Appendix A and summarized in Table 3. These data are discussed first in terms of homogeneity of variance and then in terms of strength, slump, and air content.

Mix Strength. Bartlett's test for homogeneity of variance showed that there is no significant difference between the variances of the five batches. It can be seen from Table 18 of Appendix B that the calculated value of M/C of 1.58 was much lower than the theoretical value of 9.49.

On the assumption of normality and equal variances, a one-way analysis of variance was made on the mean batch strengths. The calculated value of F from this analysis was 6.44 which is much larger than the theoretical value of 2.87 as shown in Table 19 of Appendix B. Therefore,

Table 3

PHYSICAL PROPERTIES OF CONCRETE

LOW-STRENGTH (LL) MIX

Batch Designation	Air Content Percent	Slump Inches	Average Ultimate Strength: After Oven Drying * psi	Average Ultimate Strength: After Fatigue Testing ** psi
LL 1	6.5	2-5/8	3500	3280
LL 2	6.1	2-3/8	3820	None tested***
LL 3	7.6	3-1/4	4060	3900
LL 4	7.5	3	3880	None tested
LL 5	6.2	1-1/2	3530	None tested

* Specimen age when tested was 34 days

** Specimen age ranged from 51 to 58 days

*** All specimens were used in fatigue testing there is reason to believe that the batch strengths of the LL series are not equal. In an effort to determine which batch means were different, a Newman-Kuels sequential range test was applied to the data. This test compares the difference in means with a significant range. If the difference in means is larger than the significant range it can be inferred that the means are significantly different. The Newman-Kuels test indicates that the mean of batch LL 3 is significantly different from the means of batches LL 1 and LL 5 as shown in Table 19 of Appendix B. No satisfactory explanation can be offered to account for this difference.

The average strength of the low-strength concrete was estimated to be 3,760 psi as shown in Table 22 of Appendix B. This value is the average of all of the specimens tested statically before fatigue testing in the LL series.

Air Content and Slump. Since an effort was made to control the slump and air content of the individual batch mixes, these properties are not likely to be normally distributed. For this reason only the mean, standard deviation, and the coefficient of variation have been calculated for the plastic characteristics of the mix. The coefficient of variation is the standard deviation expressed as a percentage of the mean. It is designated as C in the following table.

Property	Number of Batches	$\bar{y}$	s	C
Slump	5	2.5 in.	0.61	24%
Air Content	5	6.8%	0.64	9%

<u>Summary</u>. Most of the preceding analyses were based on the assumption that the data followed a normal distribution when actually the shape of the distribution was not known. It is felt, however, that the lack of normality, if any exists, will affect the results only slightly and that the following conclusions may be drawn:

1. The variance of the batches of low-strength concrete were not significantly different.
2. The average strength of batch LL 3 is significantly different than batches LL 1 and LL 5.
3. The best estimate of the average strength of the low-strength series is 3760 psi with a standard error of 294 psi.
4. The average slump of the low-strength series was 2.5 inches and the coefficient of variation is 24 per cent. This coefficient is much larger than the desirable value of 5 per cent usually strived for in laboratory work.
5. The average air content of the low-strength series is 6.8 per cent with a coefficient of variation of 9 per cent.

High-Strength Concrete

The data for the high-strength concrete are shown in Tables 13 through 17 of Appendix A and are summarized in Table 4. Table 4 includes the properties of batch HL 1 which are not shown in Appendix A. This batch was used to determine the effect of the rate of load application on the fatigue properties of the concrete, and will not be included in the subsequent analyses. The low-strength series was first tested for homogeneity of variance of the batch strengths and then for difference in batch mean strengths. Properties of the plastic concrete

Table 4

PHYSICAL PROPERTIES OF CONCRETE

HIGH-STRENGTH (HL) MIX

			Average Ultimate Strength	
Batch Designation	Air Content Percent	Slump Inches	After Oven Drying * psi	After Fatigue Testing ** psi
HL 1	6.8	2-3/8	5130	None tested ***
HL 2	6.6	2-1/8	6310	None tested
HL 3	6.5	2-3/8	6360	None tested
HL 4	6.3	1-7/8	6360	6180
HL 5	6.8	3	6260	None tested
HL 6	7.0	2-3/4	6010	5790

* Specimen age ranged from 44 to 69 days

** Specimen age ranged from 84 to 94 days

*** All specimens were used in fatigue testing are also discussed.

Mix Strength. Bartlett's test for homogeneity of variance indicates that there is no significant difference between the variance of the individual batches. The calculated value of M/C was 1.39 which is much smaller than the theoretical value of 9.49. Table 20 of Appendix B shows the calculations used in this test.

The one-way analysis of variance indicated that, for the high-strength concrete, no significant difference existed between the individual batch means. The calculated F value of 1.02 was considerably lower than the theoretical value of 2.87 as shown in Table 21 of Appendix B.

The average strength of the high-strength concrete was estimated to be 6,260 psi as shown in Table 22 of Appendix B. This value is the average of all specimens tested before fatigue testing of the HL series.

Air Content and Slump. As in the case of the low-strength concrete only an estimate of the mean, standard deviation, and coefficient of variation have been calculated for the slump and air content. The results of these calculations are shown in the following table.

Property	Number of Batches	$\bar{y}$	S	C
Slump	5	2.4	0.41	17%
Air Content	5	6.5	0.35	5%

Summary. The following conclusions can be drawn from the above analysis of the high-strength concrete data, assuming that the data follow a normal or nearly normal distribution:

1. The variances of the individual batches of high-strength concrete were not significantly different.
2. The average strength of the individual batches of high-strength concrete were not significantly different.
3. The best estimate of the average strength of the high-strength series is 6,260 psi with a standard error of 329 psi.
4. The average slump of the high-strength series was 2.4 inches with a coefficient of variation of 17 per cent. This coefficient is much larger than the desirable value of 5 per cent usually strived for in laboratory work.
5. The average air content of the high-strength series was 6.5 per cent with a coefficient of variation of 5 per cent.

Comparison of Series

The two mix designs were compared statistically to see if they did represent two different populations as planned. The variance of each series is given along with the series mean and the number of observations in Table 22 of Appendix B. The variances shown in this table are the total sum of squares divided by the corresponding degrees of freedom shown in Tables 19 and 21 of Appendix B multiplied by 100. The ratio of these two variances was calculated to be 1.25 which is less than the theoretical F-value of 1.98. Hence, at the 5 per cent significance level there is no reason to believe that the series variances are different.

The theoretical F-value at the 25 per cent significance level is 1.32 which is still larger than 1.25. It is, therefore, reasonable to assume that the variances are equal for the purpose of the t-test between series mean strengths. As shown in Table 22 of Appendix B, the calculated t-value was 28.299 which is much larger than the theoretical value of 2.064. The results of this analysis indicates that the two series represent two different populations, but the populations have the same variance.

By inspection it can be seen that the magnitudes of slump and air content are about the same for each series. The variation of these properties is larger in the low-strength series. The concrete in both series experienced some bleeding immediately after it was placed in the molds. It was noticed that in the high-strength series there was less bleeding than in the low-strength series. This can be expected because the low-strength concrete contained a larger proportion of coarse aggregate and was a harsher mix. Since it was planned to test concretes representing two different populations, these differences in plastic characteristics are assumed to have no significant effect on the results.

Conclusions which can be drawn from the analysis of mix data are as follows:

1. The variances of the batch strengths were not significantly different in either the high-strength or low-strength concrete.
2. The batch means were significantly different in the low-strength series but were not significantly different in the high-strength series.
3. The total variance of the low-strength series was not significantly different than the total variance of the high-strength series.

4. The average strength of the high-strength concrete was significantly different than the average strength of the low-strength concrete.
5. The slump and air content of the concrete appear to be about the same for both series. The variation of these properties between batches was greater in the LL series.
6. Concrete in both the HL series and the LL series experienced bleeding when placed in the molds. The bleeding was more severe in the LL series.

Effect of Age on Strength

Since the strength of concrete may increase with age, it seemed necessary to check for any gain in strength that may have occurred during the time required for fatigue testing. Even though all specimens were oven-dried to preclude this strength gain, it was felt that additional information should be collected.

Two batches from each series contained an adequate number of specimens remaining after fatigue testing to give a statistically sound estimate of the average batch strength. The batches used for this test were LL 1 and LL 3 in the low-strength series and HL 4 and HL 6 in the high-strength series. The second estimate of the average batch strength was made and compared to the original batch strengths in Tables 23 through 26 of Appendix B.

The variances of the individual batch strengths were first tested, and in no case did the analysis show any significant difference between the two batch variances. The estimates of the average batch strength were then compared by a t-test. The calculated t-value for batch LL 1 was 2.373 which was larger than the theoretical value of 2.306. Hence, there is a significant difference between the estimates. Observation of the data in Table 23 of Appendix B indicates that the mean strength of batch LL 1 decreased with age rather than increased. The t-tests on the other three batches indicated that there was no significant difference in the strength of the concrete before fatigue testing and the strength of the concrete after fatigue testing.

Since three of the four batches tested showed no significant increase in strength during the time of fatigue testing and the other showed a decrease of only 218 psi it seems obvious that the concrete specimens did not gain strength during the period of fatigue testing. The small difference observed in the opposite direction for a single batch is not considered large enough to give rise to significant differences in the fatigue test results.

Fatigue Test Results

Fatigue data are best represented by a plot of the stress level, expressed as a percentage of the static ultimate strength, versus the number of cycles to failure. If a curve can be fitted to this plot it is called an S-N curve. It is often difficult to fit a curve to a plot of this type because of the great amount of scatter resulting from fatigue testing.

The large amount of scatter can be explained by the very nature of fatigue. In order to obtain an estimate of the strength of any batch a number of cylinders must be tested to failure. Because of the physical variation from piece to piece, and even within pieces, the ultimate strength of any specimen is unlikely to be the same as the batch estimate. Since the static test specimens cannot be used for fatigue testing, only a crude estimate of the strength of the specimen to be tested in fatigue can be made.

It is necessary to conduct fatigue testing programs over a long period of time, during which changes in atmospheric conditions take place. Some of the scatter in a plot of stress level versus cycles to failure is undoubtedly due to errors in measuring and loading caused by these atmospheric changes.

The amount of time required to conduct a fatigue testing program also affects the amount of data which can be collected. It is impractical, from the standpoint of required time, to collect a large amount of data at the lower stress levels. To establish the existence of a fatigue limit, however, much data would be needed at the lower stress levels. Generally if a specimen endures a predetermined number of cycles without failing, it is removed from the testing machine and the next specimen is tested. In this study the maximum number of cycles which a specimen was allowed to endure was ten million.

Analysis of Fatigue Test Data

The analysis of the fatigue data is limited by the characteristics of the data which were discussed in the previous section. When analyzing the data it was necessary to assume that each specimen had the properties defined by the static tests on the corresponding batch. Other difficulties developed as the data was collected which made interpretation of the results more indefinite.

In the case of the low-strength concrete, all of the specimens at the 40 per cent stress level endured ten million repetitions of loading without failure, as shown in Table 5, and were removed from the testing machine. Thus it was impossible to determine the true fatigue life of lightweight concrete at the 40 per cent stress level from the data collected in this study. All of the specimens tested at the 50 per cent stress level failed before enduring ten million cycles except one. Specimen number two of batch LL 3 had endured 9.2 million when a power failure stopped the machine. Since this specimen endured nearly twice as many cycles as the other specimens of the LL series which were tested at the 50 per cent stress level, it was felt that if the power failure had not occurred the specimen would have endured the maximum of ten million cycles without failure.

If the assumption that specimen two of batch LL 3 endured ten million cycles without failure is accepted, the data at the 50 per cent stress level is incomplete. Using only the four specimens which failed at the 50 per cent stress level, when actually five were tested, would introduce a bias into the statistical interpretation of the fatigue data. For this reason a better approximation can be made of the S-N relationship if only the data at the 60, 70, and 80 per cent stress levels are included in the calculations.

In the case of the high-strength concrete, the first two specimens tested at the 40 per cent stress level endured the maximum of ten million cycles of loading without failure. Since all of the specimens of the LL series tested at the 40 per cent stress level endured ten million repetitions of loading, it was felt that no further information could be gained from testing the remaining scheduled specimens of the HL series at the 40 per cent stress level. At the 50 per cent stress level only two specimens failed before they had endured ten million

Table 5

FATIGUE TEST RESULTS

LOW-STRENGTH CONCRETE

Batch Designation	Specimen Number	Maximum Fatigue Load	Minimum Fatigue Load	Number of Stress Cycles Endured
LL 1	1	9,900 (40)*	900 (3.6)*	10,304,600→ **
	2	12,400 (50)	800 (3.2)	3,147,600
	3	14,800 (60)	700 (2.8)	688,700
	4	17,300 (70)	900 (3.6)	43,000
	5	19,800 (80)	800 (3.2)	19,100
LL 2	1	10,800 (40)	900 (3.3)	10,005,400→
	2	13,500 (50)	800 (3.0)	4,926,400
	3	16,200 (60)	500 (1.9)	396,600
	4	18,900 (70)	800 (3.0)	52,800
	5	21,600 (80)	800 (3.0)	3,500
LL 3	1	11,500 (40)	700 (2.4)	10,464,100→
	2	14,300 (50)	500 (1.7)	9,204,100→
	3	17,200 (60)	900 (3.1)	1,610,000
	4	20,100 (70)	800 (2.7)	51,400
	5	23,000 (80)	500 (1.7)	1,600
LL 4	1	11,000 (40)	1,000 (3.6)	10,418,100→
	2	13,700 (50)	1,100 (4.0)	5,673,300
	3	16,500 (60)	900 (3.3)	1,217,000
	4	19,200 (70)	800 (2.9)	105,100
	5	22,000 (80)	900 (3.3)	8,600
LL 5	1	10,000 (40)	1,000 (4.0)	11,723,300→
	2	12,500 (50)	1,000 (4.0)	4,628,100
	3	15,000 (60)	1,000 (4.0)	2,262,500
	4	17,500 (70)	900 (3.6)	26,300
	5	19,900 (80)	1,100 (4.3)	1,900

* Figure in parentheses is the dynamic load expressed as a percentage of the average ultimate strength of the batch.

** → indicates that the specimen had not failed when the test was stopped.

cycles of loading as shown in Table 6. Furthermore, these two specimens come from the same batch. Therefore the data collected at the 50 per cent level could not be used to interpret the S-N relationship of the HL series.

At the 60 per cent stress level all but one specimen failed before enduring the ten million limit. This specimen, specimen 3 of batch HL 3, endured about 9.7 million before it was stopped by a power failure.

If the test on this specimen had not been stopped by a power failure, the limiting number of stress cycles would probably have been reached.

Two alternatives existed for analyzing the fatigue data of the high-strength concrete. The first was to neglect the data collected at the 60 per cent stress level because of the bias introduced into the S-N relationship by considering only four test specimens when actually five were tested. If this were done, the S-N relationship would be defined from data collected at the 70 and 80 per cent stress levels. The second alternative was to include the data taken at the 60 per cent stress level knowing that a bias was being introduced. It was felt that the use of three sets of data which included a small bias in one set would give a better estimate of the true S-N relationship than two sets of data which included no bias. Therefore, the second alternative was selected.

At the 70 per cent stress level, only four specimens were tested from the HL series. This is a result of a faulty automatic shut-down on the Amsler machine which stopped the testing several times before failure had occurred. Thus the data at this stress level is incomplete but no bias is introduced here because all specimens which were tested failed.

Table 6

FATIGUE TEST RESULTS

HIGH-STRENGTH CONCRETE

Batch Designation	Specimen Number	Maximum Fatigue Load	Minimum Fatigue Load	Number of Stress Cycles Endured
HL 2	1	17,800 (40)*	1,000 (2.2)*	10,458,700 → **
	2	22,200 (50)	1,200 (2.7)	10,620,300 →
	3	26,700 (60)	1,100 (2.5)	720,500
	4	Specimen not tested		
	5	35,500 (80)	1,200 (2.7)	6,100
HL 3	1	18,000 (40)	1,100 (2.4)	10,216,500 →
	2	22,500 (50)	900 (2.0)	10,474,400 →
	3	27,000 (60)	1,000 (2.2)	9,673,500 →
	4	31,500 (70)	1,000 (2.2)	150,900
	5	36,000 (80)	1,000 (2.2)	6,600
HL 4	1	Specimen not tested		
	2	22,500 (50)	1,000 (2.2)	4,751,300
	3	22,500 (50)	1,200 (2.7)	5,957,200
	4	27,000 (60)	1,000 (2.2)	736,400
	5	31,000 (70)	1,000 (2.2)	166,100
	6	36,000 (80)	1,000 (2.2)	5,400
HL 5	1	Specimen not tested		
	2	22,200 (50)	1,300 (2.9)	10,499,400 →
	3	26,600 (60)	1,100 (2.5)	6,737,500
	4	31,000 (70)	1,100 (2.5)	611,900
	5	35,400 (80)	1,100 (2.5)	110,000
HL 6	1	Specimen not tested		
	2	21,200 (50)	1,200 (2.8)	10,196,100 →
	3	25,400 (60)	1,100 (2.6)	4,116,700
	4	29,700 (70)	1,100 (2.6)	753,600
	5	33,900 (80)	1,100 (2.6)	86,300

* Figure in parentheses is the dynamic load expressed as a percentage of the average ultimate strength of the batch.

** → indicates that the specimen had not failed when the test was stopped.

The S-N Diagrams. A semi-log coordinate system was used to plot the S-N relationship because this system presented the data in a form which is easier to view and easier to interpret than either a log-log plot or an arithmetic plot. The data collected from the LL series are plotted in Figure 9 and the data collected from the HL series are plotted in Figure 10.

When the data from the HL series were plotted there appeared to be two separate and distinct relationships. The concrete of batches HL 5 and HL 6 seemed to have different fatigue properties than the concrete of batches HL 2, HL 3, and HL 4. The reason for this separation of data is unknown. Referring to Table 4 it is seen that the slump and the air content of batches HL 5 and HL 6 are larger than the same properties of batches HL 2, HL 3, and HL 4. A correlation between slump and air content can be expected but it seems unlikely that the fatigue properties would be altered by such small variations in the slump and air content. The scatter of data within the two apparent sets of data is not great enough to substantiate any correlation between fatigue properties and plastic properties of the high-strength concrete. The only conclusion that can be drawn from the data collected in this study is that the fatigue properties of the high-strength concrete are somewhat influenced by the slump, the air content, or both.

The S-N diagrams indicate that no fatigue limit was reached at ten million cycles of repeated loading for the concrete tested in this study. None of the S-N curves appear to become horizontal nor even start to level out in the neighborhood of ten million cycles.

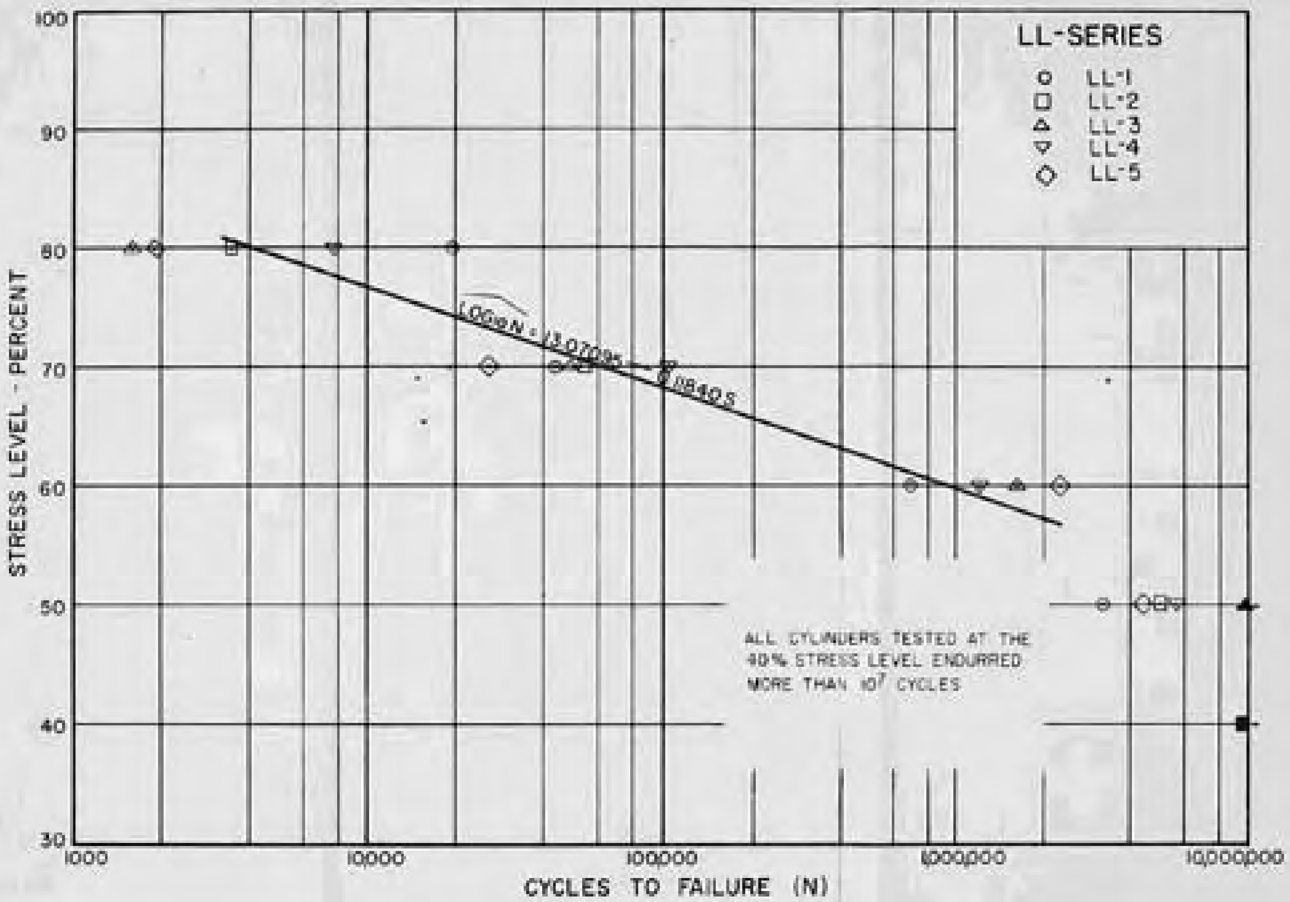

FIGURE 9. S-N DIAGRAM FOR LOW STRENGTH CONCRETE

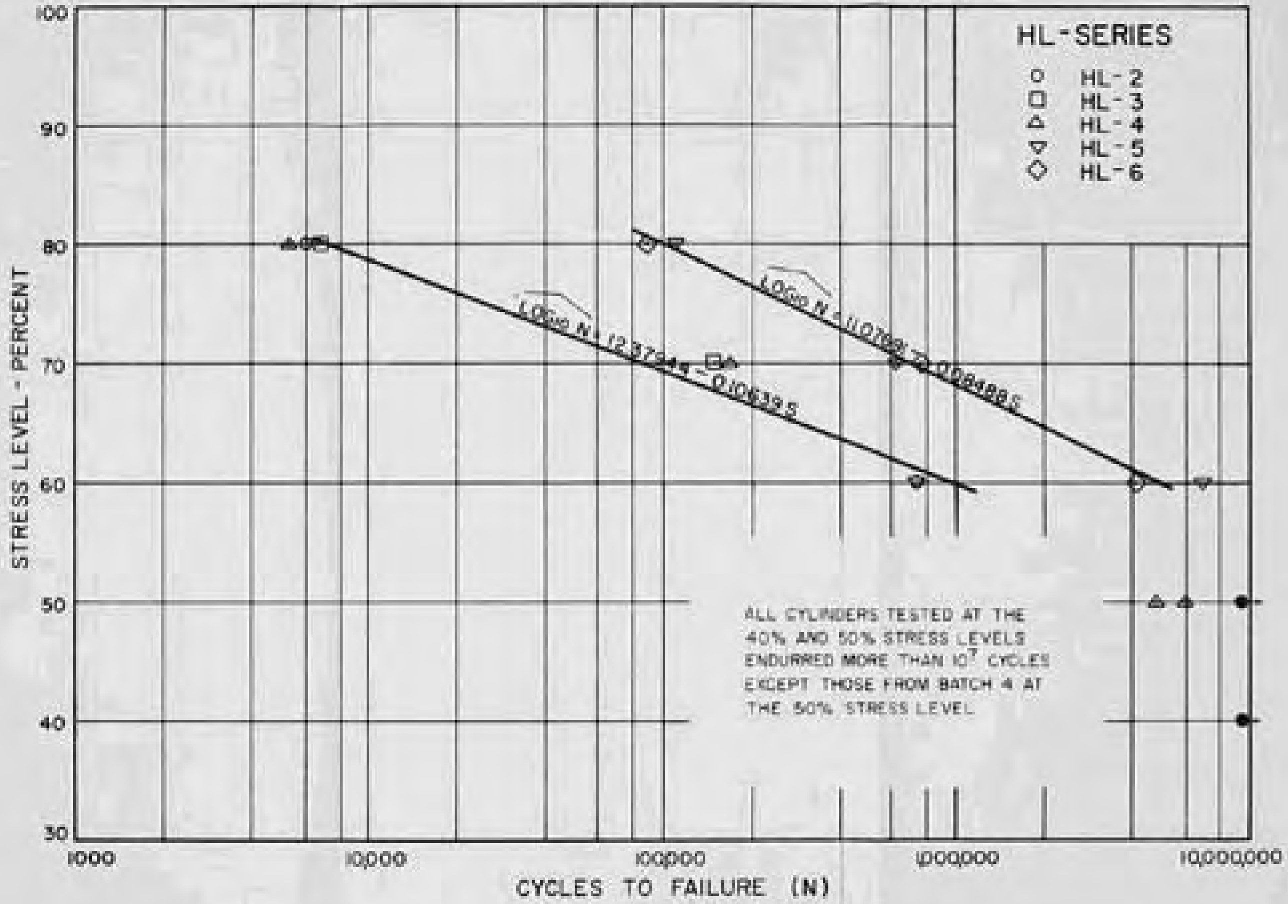

FIGURE 10. S-N DIAGRAM FOR HIGH STRENGTH CONCRETE

<u>Comparison of the Two Mixes</u>. Since the form of the distribution of fatigue data was unknown, it was felt that some test should be employed to determine whether or not the two sets of data represent populations having the same frequency distribution. A method called "Runs" was chosen for this purpose (21). When using this test one can infer that if there are no more or no fewer runs in the data than would be expected by chance, the populations have the same frequency distribution. The run test was applied to the data of each stress level and the calculations are shown in Table 27 of Appendix C.

At the 60, 70, and 80 per cent stress levels, the run test shows that there is no reason to believe the two populations have different frequency distributions. Tests could not be made at the 40 and 50 per cent stress level because many of the specimens endured ten million cycles of loading and were removed from the testing machine before failure. Hence, there is no way of estimating the number of cycles these specimens would have endured if the tests had been continued. A similar condition existed at the 60 per cent stress level, but it was felt that the run test could be applied to those specimens which failed at fewer than ten million cycles.

<u>Linear Regression</u>. The simplest type of regression is a linear regression. If the data to be analyzed does not follow a linear relationship, they can often be transformed by extracting the square root, squaring, or taking the logarithm of the individual values. By taking the logarithm of the individual values of the cycles to failure, the data of this study appeared to have a linear S-N relationship (Figures 9 and 10). Linear regression calculations were based on the assumption

that the S-N relationship was linear and that the transformed data were normally distributed.

Three curves were fitted to the fatigue test data, one for the LL series and two for the HL series. For purposes of calculation, the LL series will be referred to as sample I. Batches HL 2, HL 3, and HL 4 will be referred to as sample II and batches HL 5 and HL 6 will be referred to as sample III. Curves were fitted to each set of data by the method of least squares. An analysis of variance was also included in the least squares computations to determine whether or not the S-N relationship was actually linear. The linear regression calculations are shown in Tables 28, 29, and 30 of Appendix C.

The least squares linear regression equation obtained from the data of sample I is:

$$\widehat{\log N} = 13.077 - 0.118S$$

where N is the number of cycles to failure and S is the stress expressed as a percentage of the static ultimate stress. This equation can be used to estimate N only when the value S lies between 60 and 80 per cent because this is the range of S used in the calculations. The calculated F in the test for linearity was 0.51 which is less than the theoretical value of 4.75. Therefore, there is no reason to believe that the S-N relationship is not linear.

The least squares linear regression equation obtained from the data of sample II is:

$$\widehat{\log N} = 12.379 - 0.106S$$

The use of this equation is also limited to values of S between 60 and 80 per cent. The calculated F value in the test for linearity was 169.95 which is much larger than the theoretical value of 7.71. There is, therefore, a significant reason to believe that the S-N relationship of these data is not linear. By visually inspecting the data of sample II in Figure 10, it can be seen that a curve passing through the means of the three stress levels would be concave downward.

Additional data would be needed to better define this curve.

The least squares linear regression equation obtained from the data of sample III is:

$$\widehat{\log N} = 11.077 - 0.0858$$

The use of this equation is limited to values of S between 60 and 80 per cent. The calculated F in the test for linearity was 22.0 as compared with the theoretical value of 10.1. Thus there is reason to believe that the S-N relationship of these data is not linear. By visually inspecting the data of sample III, Figure 10, it can be seen that a curve passing through the means of the three stress levels would be concave upwards. This indicates that possibly a fatigue limit is being approached.

Additional data would be needed to better define this curve.

Calculations for correlation coefficients were also included in the linear regression analyses. In all three samples it was found that there is a very high degree of association between cycles to failure and the stress level. The correlation coefficients were found to be -0.952, -0.982, and -0.956 for samples I, II, and III respectively. Table 31 of Appendix C shows the calculations for a test to determine whether the correlation coefficients are significantly different and the procedure is outlined in reference (22). The test showed that there is no reason to believe that the correlation coefficients are different. Hence, the degree of association between the stress level and the number of cycles to failure can be assumed to be the same for all three samples.

Comparison of S-N Curves. In the comparison of the three S-N curves it is necessary to first consider the data of the HL series. These data appear to be separated into two separate and distinctive S-N relationships for no apparent reason. If there is a specific reason for this separation it was not planned in the design of the concrete tested and, therefore, a correction to the usual statistical methods must be made.

It is necessary to determine the number of ways in which the five batches of concrete could have been divided into two main groups. There are ten ways in which the five batches could have been divided so that one group contained two batches and the other contained three batches. There are also five ways in which the batches could have been divided so that one group contained four batches and the other group contained only one batch. Hence, there is a total of 15 ways in which the data of the HL series could have been placed into two separate groupings.

If the characteristics of these two groups were to be compared by using an F, or any other statistic, at the 5 per cent significance level by the usual statistical procedures, the actual statistical significance level would be 15 times 5 per cent or 75 per cent. It is evident, therefore, that if it is desired to compare these two curves at any given significance level, the given significance level must be divided by 15 to produce reliable results.

Since it is desired to compare the two curves of the HL series with the curve of the LL series a factor other than 1/15 must be applied to the desired significance level because the five batches of the LL series have not been separated into two groups. At the present time, there is no method available to determine the exact reduction factor which should be used. It is felt, however, that the proper reduction factor for the three sets of data under consideration is probably between 1/10 and 1/15. The following tests on slopes and intercepts were conducted at the apparent significance level of 1/2 per cent on the assumption that the reduction factor of 1/10 would yield sufficiently accurate results for the purposes of this calculation. The fact that the degrees of freedom used in these tests are relatively large, tends to verify this conclusion because the difference between test statistics decreases as the degrees of freedom increase.

An analysis of covariance was used to determine whether the slopes of the three regression curves were significantly different. The computations for this analysis are shown in Table 32 of Appendix C and are described in reference (19). The results of this test indicate that there is no reason to believe that the slopes of the three regression equations are different. The analysis does show reason to believe that the slopes of the three regression equations are not the same as a common over-all slope. It is also apparent that the slopes within samples do not differ significantly from the slope between samples.

A two-way analysis of variance test was conducted on the combined data of all three samples to see if there was any difference in the intercepts of the regression equation. This analysis is shown in Table 33 of Appendix C and indicates that there is a significant difference between intercepts. A Newman-Kuels sequential range test showed that the intercept of sample III was different than the intercepts of the other two samples.

> Since the slope is the property which best defines the S-N relationship, it seems reasonable to conclude that the fatigue properties of the three sets of data are the same.

In an effort to strengthen this conclusion the prediction intervals for each sample were calculated. The calculations are shown in Table 34 of Appendix C and summarized in Table 7. It was felt that if the prediction intervals overlapped, it could be definitely concluded that the fatigue properties of all three samples were the same.

Referring to Table 7 it is clear that the linear regression equation for sample I lies within the prediction intervals calculated for sample II. It is also evident that the linear regression equation of sample II lies within the prediction intervals of sample I. Therefore, there is no reason to believe that the fatigue properties of sample I and sample II are different. On the basis of the prediction interval concept, the data of sample III appears to be separated from that of the other two samples.

The relationship between the stress level and the number of cycles

Table 7

NINETY-FIVE PER CENT PREDICTION INTERVALS

Prediction Intervals Cycles to Failure

Stress Level Per cent	Sample I Lower Limit	Sample I Upper Limit	Sample II Lower Limit	Sample II Upper Limit	Sample III Lower Limit	Sample III Upper Limit
60%	4.786×10^5	1.845×10^6	4.575×10^5	2.146×10^6	1.691×10^6	1.449×10^7
70%	4.014×10^4	9.360×10^4	5.390×10^4	1.357×10^5	3.896×10^5	1.383×10^6
80%	2.051×10^3	7.906×10^3	3.842×10^3	1.419×10^4	3.392×10^4	2.906×10^5

Estimated Cycles to Failure

Stress Level Per cent	Sample I	Sample II	Sample III
60%	9.396×10^5	9.909×10^5	4.949×10^6
70%	6.151×10^4	8.553×10^4	7.010×10^5
80%	4.027×10^3	7.383×10^3	9.998×10^4

to failure is represented most clearly by the slope of the linear regression equation. The results of the analysis of slopes suggest that the data of all three samples have the same slope and therefore suggest that all three samples have the same S-N relationship. The prediction intervals tend to weaken the conclusion that all three samples have the same S-N relationship but this test is not as strong as the analysis of slopes because individual errors rather than a total combined error is used. It is, therefore, felt that the strongest and most likely conclusion that can be drawn from this analysis is that the fatigue properties of all concrete tested in this study are the same regardless of differences in the static properties.

Type of Failure

It was not possible to observe the process of fatigue failure because all of the specimens that failed, did so extremely rapidly and without warning. The appearnace of specimens that failed in fatigue was not unlike the appearance of those which failed statically. Typical failures of static and fatigue specimens are shown in Figure 11. Automatic shut-down devices were employed on both machines so that no crushing load was applied to the failed specimens.

Other Observations

It was noticed that a change in load occurred on most fatigue specimens during the last few thousand cycles before failure. This change consisted of a decrease in maximum load and an increase in minimum load. This indicates that as the specimen nears the failing point, a change takes place in the stress-deformation properties of the concrete which is reflected back into the characteristics of the fatigue machine and its recording device.

FIGURE 11. TYPICAL FAILURES

Speed of Testing Results

Since two machines were used that applied loads at different rates, it was felt that some testing should be done to determine if the different rate of load application had any effect on the fatigue properties of the concrete. Batch LL 1 was used for this purpose.

Nine specimens were tested in each machine at the 80 per cent stress level and a t-test was made on the means obtained. Specimens tested in the Krouse-Purdue machine were tested at a rate of 1,000 cycles per minute and those tested in the Amsler machine were tested at a rate of 500 cycles per minute.

An F-test was used to compare the variance of the number of cycles to failure for each machine. As shown in Table 35 of Appendix C the calculated F was 2.015 and the theoretical F was 3.44. Therefore, there is no reason to believe that the variances of the two sets of data were different. The t-test was then conducted and the calculated t-value of 0.264 was compared with the theoretical value of 2.12. Since $2.12 > 0.264$ there is no reason to believe that the two means are different.

It seems reasonable to conclude that the speed of test did not affect the fatigue properties of lightweight aggregate concrete when the speed of testing was between 500 and 1,000 cycles per minute. This conclusion is in accordance with the statements expressed in the available literature on the rate of testing of concrete fatigue specimens (15, 23).

Summary of Fatigue Test Results

Throughout this study care was taken to minimize the differences between batches. An attempt was made to make all of the concrete of each series as nearly the same as possible with respect to strength, air content, and consistency. All specimens were prepared for testing in an identical manner. Even with this attempt to control the physical properties of the concrete and the test procedures, the fatigue test data showed a large amount of scatter. Some of this scatter was undoubtedly due to variations in atmospheric conditions because the fatigue tests were begun in summer and continued through most of the winter. The largest part of the scatter is accounted for, however, by the observed variations in physical properties between batches and within individual specimens.

The results obtained from the analysis of fatigue test data may be summarized as follows:

1. The linear regression equation of the S-N relationships for the low-strength concrete for values of S between 60 and 80 per cent is:

$$\widehat{\log N} = 13.077 - 0.118\ S$$

2. The data of the high-strength series appeared to form two separate S-N relationships. The linear regression equations of these two S-N relationships for values of S between 60 and 80 per cent are:

$$\widehat{\log N} = 12.379 - 0.106\ S$$

and

$$\widehat{\log N} = 11.077 - 0.085\ S$$

3. There is no reason to believe that a difference exists between the fatigue properties of low-strength and the

high-strength concrete. Hence there is no reason to believe that the gradations used in this study have any effect on the fatigue properties of lightweight aggregate concrete.

4. The S-N diagrams appear to show no fatigue limit for lightweight aggregate in the neighborhood of ten million repetitions of loading.

5. There is no reason to believe, from the results obtained on the 18 specimens tested, that the rate of load application has an effect on the fatigue properties of lightweight aggregate concrete when the rate of load application lies between the values of 500 and 1,000 cycles per minute.

Comparison of Lightweight Concrete With Normal Weight Concrete

In July of 1958, the results of a study comparing the fatigue properties of air-entrained concrete with the fatigue properties of non-air-entrained concrete was reported (13). That study was made in the same laboratory as the present study. The testing programs were nearly identical except for the materials used. The aggregate utilized in the 1958 study was a crushed limestone from Central Indiana. The cement used in both studies was from one clinker batch (laboratory designation 315) and is assumed to have the same properties.

It was felt that since the conditions under which these two studies were conducted were similar, a comparison could be made to determine whether the fatigue properties of lightweight concrete are different than the fatigue properties of normal weight concrete. A comparison of correlation coefficients is shown in Table 36 of Appendix D and a significance test for difference between slopes is shown in Table 37 of Appendix D. The values used for these tests can be found in Appendix C of Reference (13). No attempt was made to compare the difference between intercepts because the values of stress level in the study on normal weight concrete varied so greatly that only one reading was recorded at some stress levels. When a cell contains only one reading a poor estimate of the error variance results and it was felt that such a test would not yield accurate results.

A reduction factor must be applied to the significance level in the test for differences in slopes. In this case a comparison of three sets of data with the two of the HL series would require a smaller reduction factor than 1/15. The actual reduction factor should be slightly less than the one used in the section on Comparison of the S-N Data. It was felt that the actual reduction factor should be about 1/10 and that this value would yield sufficiently accurate results. The tests were, therefore, conducted at the 1/2 per cent significance level to detect any difference in slopes at the 5 per cent significance level. The test indicated that at the 5 per cent significance level there is no reason to believe that the slopes are different.

Since the slope is the property which best defines the S-N relationship, it seems reasonable to conclude that the fatigue properties of the concrete used in the present study do not differ from the fatigue properties of normal concrete. Hence a reduction in dead load can be attained for all types of construction using lightweight aggregate concrete, and the design strength would not need to be increased above that required for normal concrete to account for repetitive loading. This result should be of particular importance to agencies engaged in the design and construction of bridges because experience has shown that substantial savings can be accrued by the use of lightweight aggregate concrete as a paving material for highway bridges.

The test for difference in correlation coefficients indicated that there is a significant difference between the correlation coefficients of the five S-N relationships tested. By comparing these coefficients visually it can be seen that the correlation coefficient of the non-air-entrained series reported in Reference (13) is about 30 per cent lower than the other four values compared. This comparison indicates that the degree of association between stress level and number of cycles to failure is much higher for concrete containing entrained air.

Antrim's findings indicate that air-entrained concrete yields more consistent fatigue data than non-air-entrained concrete (13). The comparison of the correlation coefficients found in this study with those found by Antrim tend to strengthen this conclusion. Hence if air-entrained concrete (lightweight or normal) is used where fatigue action is anticipated, the relationship between cycles to failure and stress level can be estimated with more confidence and, therefore, a lower factor of safety could be employed. Thus the entrainment of air can be beneficial to the resistance of repetitive loading as well as durability and workability.

SUMMARY OF RESULTS

In this section, the significant results discussed in the foregoing sections are restated in the order in which they were considered;

1. The average strength of the low-strength concrete was 3,760 psi and the average strength of the high-strength concrete was 6,260 psi. The variance of the batch strengths within each of these series was not significantly different but the average of each series did differ significantly.

2. The average slump of the low-strength series was 2.5 inches with a coefficient of variation of 24 per cent. The average slump of the high-strength series was 2.4 inches with a coefficient of variation of 17 per cent.

3. The air contents of the low and high strength concretes were 6.8 and 6.5 per cent respectively. The coefficients of variation were 9 and 5 per cent respectively.

4. Both mix designs experienced bleeding at the time of casting. The bleeding of the low-strength concrete was more noticeable than that of the high-strength concrete.

5. There was no significant change in the ultimate compressive strength of the concrete during the time of fatigue testing.

6. The S-N relationship of the low-strength concrete between stress levels of 60 and 80 per cent, as determined by a linear regression analysis was:

$$\widehat{\log N} = 13.077 - 0.118\ S$$

7. The fatigue data of the high-strength concrete appeared to have two separate and distinct S-N relationships. The linear regression equations for these two relationships between the stress levels of 60 and 80 per cent were:

$$\widehat{\log N} = 12.379 - 0.106\ S$$

and

$$\widehat{\log N} = 11.077 - 0.085\ S.$$

8. The degree of association between the number of cycles to failure and the stress level was the same for the three S-N relationships investigated in this study. It was found that the slopes of the three S-N curves had the same slope but that the intercepts of one of the high-strength concrete relationships differed from the other two intercepts. Prediction intervals also indicated that the S-N curve with the different intercept was separated from the other two relationships. Since the slope of the S-N curve best describes the relationship between stress level and cycles to failure, it was concluded that the S-N relationship was the same for both the low-strength and high-strength concrete.

9. The fatigue data collected in this study indicated that in general the S-N relationship did not tend to level out and,

therefore, there is no fatigue limit for lightweight aggregate concrete in the neighborhood of ten million repetitions of loading.

10. Tests on 19 specimens indicated that there is no difference in the fatigue properties when the rate of load application varies between 1,000 and 500 cycles per minute.

11. Comparison of the results of this study and the results of a previous study, which compared the fatigue properties of air-entrained and non-air-entrained concrete, indicated that there is no difference between the fatigue properties of lightweight aggregate concrete and normal weight concrete.

CONCLUSIONS

The following conclusions can be drawn from the fatigue data collected in this study. These conclusions are based on the fatigue testing of forty-seven 3 inch by 6 inch lightweight aggregate concrete specimens tested in direct compression at a rate of loading which varied from 1,000 to 500 cycles per minute.

1. The fatigue properties of lightweight aggregate concrete are not changed significantly by varying the proportions of fine and coarse aggregate in the mix design.

2. The fatigue properties of lightweight aggregate concrete are not significantly different than the fatigue properties of normal weight concrete.

SUGGESTIONS FOR FURTHER WORK

This study has been concerned only with the fatigue properties of lightweight concrete having two different strengths. The amount of time required to collect data limited the range of stress level over which fatigue data could be collected. Unfortunately many studies on the fatigue of concrete will probably be hampered by the time factor unless the investigators have several years in which to investigate the fatigue of concrete.

The results of this study indicate that every precaution must be taken to minimize the number of variables which may be acting during an investigation. Care should be taken to test specimens at, as nearly as possible, the same age and any other control that would eliminate the batch-to-batch differences should be employed.

Following is a list of suggestions for further research:

1. More information is needed to supplement the results of this thesis. Tests should be conducted at lower stress levels to more fully investigate the existence of fatigue limit for lightweight aggregate concrete.

2. The literature has indicated that the moisture content of lightweight aggregate immediately prior to mixing has no effect on the strength of the concrete. The moisture content may, however, have an effect on the fatigue properties of the concrete and this variable should be investigated.

3. More tests are needed to determine the effect of the rate of fatigue load application on the fatigue properties. If it can be definitely concluded that the rate of load application has no effect on the fatigue of concrete, more rapid testing would enable future investigators to more completely determine the effects of the variables which they are concerned with. Temperature measurements should also be included in this type of testing.

4. Tests should be conducted to determine the mechanism in which fatigue failures take place in concrete. This would shed more light on previous investigations and also aid future experimenters.

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APPENDIX A

TEST DATA FOR CONCRETE MIXES

Table 8

DATA SHEET FOR LL MIX - Batch 1

Plastic Properties

slump = 2-5/8 inches

air content = 6.5 per cent

Curing

1 day in molds

27 days in saturated lime solution

Drying

age at start = 28 days

age at finish = 32 days

Capping

at age of 33 days

Static compression test

age when tested (days)	34	58
breaking stress (psi)	3,500	3,270
	3,570	3,250
	3,110	3,470
	3,560	3,400
	3,750	3,020

Fatigue tests

age of specimens: at start = 34 days

at finish = 55 days

Minimum Load (lb)	Maximum Load (lb)	Stress Cycles Endured	Per cent of Static Ultimate Strength
900	9,900	10,304,600	40
800	12,400	3,147,600 failed	50
700	14,800	688,700 failed	60
900	17,300	43,000 failed	70
800	19,800	19,100 failed	80

Table 9

DATA SHEET FOR LL MIX - Batch 2

Plastic Properties

slump = 2-3/8 inches

air content = 6.1 per cent

Curing

1 day in molds

27 days in saturated lime solution

Drying

age at start = 28 days

age at finish = 32 days

Capping

at age of 33 days

Static compression test

age when tested (days)	34
breaking strength (psi)	3,800
	3,450
	3,880
	4,210
	3,760

Fatigue tests

age of specimen: at start = 34 days

at finish = 45 days

Minimum Load (lb)	Maximum Load (lb)	Stress Cycles Endured	Per cent of Static Ultimate Strength
900	10,800	10,005,400	40
800	13,500	4,926,400 failed	50
500	16,200	396,600 failed	60
800	18,900	52,800 failed	70
800	21,600	3,500 failed	80

Table 10

DATA SHEET FOR LL MIX - Batch 3

Plastic Properties

slump = 3-1/4 inches

air content = 7.6 per cent

Curing

1 day in molds

27 days in saturated lime solution

Drying

age at start = 28 days

age at finish = 32 days

Capping

at age of 33 days

Static Compression Test

age when tested (days)	34	51
breaking strength (psi)	3,920	3,970
	4,060	4,060
	4,240	3,810
	4,200	3,850
	3,890	3,800

Fatigue Tests

age of specimen: at start = 36 days

at finish = 51 days

Minimum Load (lb)	Maximum Load (lb)	Stress Cycles Endured	Per cent of Static Ultimate Strength
700	11,500	10,464,100	40
500	14,300	9,204,100	50
900	17,200	1,610,000 failed	60
800	20,100	51,400 failed	70
500	23,000	1,600 failed	80

Table 11

DATA SHEET FOR LL MIX - Batch 4

Plastic Properties

slump = 3 inches

air content = 7.5 per cent

Curing

1 day in molds

27 days in saturated lime solution

Drying

age at start = 28 days

age at finish = 32 days

Capping

at age of 33 days

Static Compression Test

age when tested (days)	34
breaking strength (psi)	3,720
	3,640
	4,160
	4,080
	3,850

Fatigue Test

age of specimen: at start = 34 days

at finish = 47 days

Minimum Load (lb)	Maximum Load (lb)	Stress cycles endured	Per cent of Static Ultimate Strength
1000	11,000	10,418,100	40
1100	13,700	5,673,300 failed	50
900	16,500	1,217,000 failed	60
800	19,200	105,100 failed	70
900	22,000	8,600 failed	80

Table 12

DATA SHEET FOR LL MIX - Batch 5

Plastic Properties

slump = 1-1/2 inches

air content = 6.2 per cent

Curing

1 day in molds

27 days in saturated lime solution

Drying

age at start = 28 days

age at finish = 32 days

Capping

at age of 33 days

Static Compression Test

age when tested (days)	34
breaking strength (psi)	3,500
	3,310
	3,600
	3,510
	3,710

Fatigue Test

age of specimen: at start = 34 days

at finish = 54 days

Minimum Load (lb)	Maximum Load (lb)	Stress cycles endured	Per Cent of Static Ultimate Strength
1000	10,000	11,723,300	40
1000	12,500	4,628,100 failed	50
1000	15,000	2,262,500 failed	60
900	17,500	26,300 failed	70
1100	19,900	1,900 failed	80

Table 13

DATA SHEET FOR HL MIX - Batch 2

Plastic Properties

slump = 2-1/8 inches

air content = 6.6 per cent

Curing

1 day in molds

27 days in saturated lime solution

Drying

age at start = 28 days

age at finish = 32 days

Capping

at age of 33 days

Static Compression Test

age when tested (days)	57
breaking stress (psi)	5,980
	6,650
	6,290
	6,480
	6,140

Fatigue Test

age of specimen: at start = 58 days

at finish = 72 days

Minimum Load (lb)	Maximum Load (lb)	Stress cycles endured	Per cent of Static Ultimate Strength
1000	17,800	10,458,700	40
1200	22,200	10,620,300	50
1100	26,700	720,500 failed	60
Specimen not tested			70
1200	35,500	6,100 failed	80

Table 14

DATA SHEET FOR HL MIX - Batch 3

Plastic Properties

slump = 2-3/8 inches

air content = 6.5 per cent

Curing

1 day in molds

27 days in saturated lime solution

Drying

age at start = 28 days

age at finish = 32 days

Capping

at age of 33 days

Static Compression Test

age when tested (days)	47
breaking stress (psi)	5,950 6,860 6,670 6,270 6,050

Fatigue Test

age of specimen: at start = 48 days

at finish = 79 days

Minimum Load (lb)	Maximum Load (lb)	Stress cycles endured	Per cent of Static Ultimate Strength
1100	18,000	10,216,500	40
900	22,500	10,474,400	50
1000	27,000	9,673,500	60
1000	31,500	150,900 failed	70
1000	36,000	6,600 failed	80

Table 15

DATA SHEET FOR HL MIX - Batch 4

Plastic Properties

slump = 1-7/8 inches

air content = 6.3 per cent

Curing

1 day in molds

27 days in saturated lime solution

Drying

age at start = 28 days

age at finish = 32 days

Capping

at age of 33 days

Static Compression Test

	69	94
age when tested (days)	69	94
breaking stress (psi)	6,260	5,830
	6,250	6,310
	6,770	6,340
	6,250	6,280
	6,250	

Fatigue Test

age of specimen: at start = 70 days

at finish = 92 days

Minimum Load (lb)	Maximum Load (lb)	Stress cycles endured	Per cent of Static Ultimate Strength
	Specimen not tested		
1000	22,500	4,751,300 failed	40
1200	22,500	5,957,200 failed	50
1000	27,000	736,400 failed	60
1000	31,000	166,100 failed	70
1000	36,000	5,400 failed	80

Table 16

DATA SHEET FOR HL MIX - Batch 5

Plastic Properties

slump = 3 inches

air content = 6.8 per cent

Curing

1 day in molds

27 days in saturated lime solution

Drying

age at start = 28 days

age at finish = 32 days

Capping

at age of 33 days

Static Compression Test

age when tested (days)	44
breaking stress (psi)	6,430 6,490 6,060 6,560 5,770

Fatigue Test

age of specimen: at start = 45 days

at finish = 89 days

Minimum Load (lb)	Maximum Load (lb)	Stress cycles endured	Per cent of Static Ultimate Strength
1300	22,200	10,499,400	50
1100	26,600	6,737,500 failed	60
1100	31,000	611,900 failed	70
1100	35,400	111,000 failed	80

Table 17

DATA SHEET FOR HL MIX - Batch 6

Plastic Properties

slump = 2-3/4 inches

air content = 7 per cent

Curing

1 day in molds

27 days in saturated lime solution

Drying

age at start = 28 days

age at finish = 32 days

Capping

at age of 33 days

Static Compression Test

age when tested (days)	69	84
breaking stress (psi)	6,000	5,720
	6,270	6,100
	6,260	6,040
	5,440	5,280
	6,260	5,640

Fatigue Test

age of specimen: at start = 71 days

at finish = 82 days

Minimum Load (lb)	Maximum Load (lb)	Stress cycles endured	Per Cent of Static Ultimate Strength
1200	21,200	10,196,100	50
1100	25,400	4,116,700 failed	60
1100	29,700	753,600 failed	70
1100	33,900	86,300 failed	80

APPENDIX B

STATISTICAL ANALYSIS OF STATIC TEST DATA

Table 18

BARTLETT'S TEST FOR HOMOGENEITY OF VARIANCE

FOR STRENGTH DATA LOW-STRENGTH CONCRETE

Coding: $Y_i = \dfrac{X_i - 3680}{10}$ where X_i is the value of individual specimen strengths recorded in Tables 8 through 12 of Appendix A.

Batch Designation

	LL 1	LL 2	LL 3	LL 4	LL 5
	-18	+12	+24	+ 4	-17
	-11	-23	+38	- 4	-37
	-57	+20	+56	+48	- 8
	-12	+53	+52	+40	-17
	+ 7	+ 8	+21	+17	+ 3
N_j	5	5	5	5	5
$\sum_i Y_{ij}$	-91	+70	+191	+105	-76
$\sum_i Y_{ij}^2$	3887	3946	8301	4225	2020
$(\sum_i Y_{ij})^2$	8281	4900	26481	11025	5776
$\bar{Y}_j$	-18.2	+14.0	+38.2	+21.0	-15.2
s_j^2	4444.70	4687.50	8552.20	4730.00	2236.20

where

$$s_j^2 = \frac{N_j}{N_j - 1}\left[\sum_i \frac{Y_{ij}^2}{N_j} - \left(\sum_i \frac{Y_{ij}}{N_j}\right)^2\right]$$

$$\sum_i N_j \log s_j^2 = 73.10092$$

$$\left[\sum_i N_j\right]\left[\log \frac{\sum_j N_j s_j^2}{\sum_j N_j}\right] = 73.85720$$

Table 18 (continued)

$$M = 2.30259\left[\left(\sum_j N_j\right)\left(\log \frac{\sum_j N_j\, S_j^2}{\sum_j N_j} \right) - \sum_j N_j \log S_j^2 \right]$$

$$= 1.7410$$

$$C = 1 + \frac{1}{3(J-1)}\left[\sum_j \frac{1}{N_j} - \frac{1}{\sum_j N_j} \right] = 1.10000$$

$$M/C = 1.58$$

$$\chi^2_{.05}(4) = 9.49 > M/C$$

At the 5% significance level there is no reason to believe that the variance of the five batch strengths are different.

Table 19

ANALYSIS OF VARIANCE FOR DIFFERENCE BETWEEN

BATCH MEAN STRENGTHS OF LOW-STRENGTH CONCRETE

$$SS_B = \sum_j \frac{(\sum_j Y_{ij})^2}{N_j} - \frac{(\sum_j \sum_i Y_{ij})^2}{\sum_j N_j} = 11708.56$$

$$SS_e = \sum_j \sum_i Y_{ij}^2 - \sum_j \frac{(\sum_i Y_{ij})^2}{N_j} = 9086.40$$

$$SS_t = \sum_j \sum_i Y_{ij}^2 - \frac{(\sum_j \sum_i Y_{ij})^2}{\sum N_j} = 20794.96$$

ANOVA TABLE

Source	df	Sum of Squares	Mean Square	F ratio
batch	4	11,708.56	2,927.14	6.44
error	20	9,086.40	454.32	
Total	24	20,794.96		

$$F_{.05}(4, 20) = 2.87 < 6.44$$

At the 5% significance level there is reason to believe that the five average batch strengths are different.

Comparison of Difference

$S_e^2 = 454.32$

$S\bar{y} = \sqrt{454.32/4} = 10.66$

$R_5 = 4.23 \times 10.66 = 45.0$

$R_4 = 3.96 \times 10.66 = 42.2$

$R_3 = 3.58 \times 10.66 = 38.2$

$R_2 = 2.92 \times 10.66 = 31.1$

Table 19 (continued)

Batch	1	5	2	4
3	56.4	53.4	24.2	17.2
4	39.2	36.2	7.0	
2	32.2	29.2		
5	3.0			

At the 5% significance level there is reason to believe that the average strength of batch LL 3 is different than the average batch strengths of batches LL 1 and LL 5.

Table 20

BARTLETT'S TEST FOR HOMOGENEITY OF VARIANCE

FOR STRENGTH DATA HIGH-STRENGTH CONCRETE

Coding: $Y_i = \dfrac{X_i - 6000}{10}$ where X_i is the value of the individual specimen strengths recorded in Tables 13 through 17 of Appendix A.

Batch Designation

	HL 2	HL 3	HL 4	HL 5	HL 6
	- 2	- 5	+26	+43	- 1
	+65	+86	+25	+49	+27
	+29	+67	+78	+ 6	+27
	+48	+27	+25	+56	-65
	+14	+ 5	+25	-23	+13
N_j	5	5	5	5	5
$\sum_i Y_{ij}$	154	180	179	131	1
$\sum_i Y_{ij}^2$	7570	12664	8635	7951	5853
$(\sum_i Y_{ij})^2$	23716	32400	32041	17161	1
$\bar{Y}_j$	30.80	36.00	35.80	26.20	0.20
s_j^2	706.7	1546.0	556.7	1129.7	1463.2

where

$$s_j^2 = \frac{N_j}{N_j - 1}\left[\sum_i \frac{Y_{ij}^2}{N_j} - \left(\sum_i \frac{Y_{ij}}{N_j}\right)^2\right]$$

$$\sum_j N_j \log s_j^2 = 60.00932$$

$$\left[\sum_j N_j\right]\left[\log \frac{\sum_j N_j s_j^2}{\sum_j N_j}\right] = 60.67240$$

Table 20 (continued)

$$M = 2.30259 \left[(\sum_j N_j) \left(\log \frac{\sum_j N_j S_j^2}{\sum_j N_j} \right) - \sum_j N_j \log S_j^2 \right]$$

$$= 1.52680$$

$$C = 1 + \frac{1}{3(j-1)} \left[\sum_j \frac{1}{N_j} - \frac{1}{\sum_j N_j} \right] = 1.10000$$

$$M/C = 1.39$$

$$\chi^2_{.05}(4) = 9.49 > M/C$$

At the 5% significance level there is no reason to believe that the variance of the five batch strengths is different.

Table 21

ANALYSIS OF VARIANCE FOR DIFFERENCE BETWEEN

BATCH MEAN STRENGTHS OF HIGH-STRENGTH CONCRETE

$$SS_B = \sum_j \frac{(\sum_j Y_{ij})^2}{N_j} - \frac{(\sum_j \sum_i Y_{ij})^2}{\sum_j N_j} = 4422.80$$

$$SS_e = \sum_j \sum_i Y_{ij}^2 - \sum_j \frac{(\sum_i Y_{ij})^2}{N_j} = 21609.20$$

$$SS_t = \sum_j \sum_i Y_{ij}^2 - \frac{(\sum_j \sum_i Y_{ij})^2}{\sum_j N_j} = 26032.00$$

ANOVA TABLE

Source	df	Sum of Squares	Mean Square	F ratio
batch	4	4422.80	1105.70	1.02
error	20	21609.20	1080.46	
Total	24	26032.00		

$F_{.05}(4,20) = 2.87 > 1.02$

At the 5% significance level there is no reason to believe that the five average batch strengths are different.

Table 22

TEST FOR DIFFERENCE OF MIX STRENGTHS

LL Mix	$N = 25$	$\bar{Y} = 3760$	$S^2 = 866.45666$
HL Mix	$N = 25$	$\bar{Y} = 6260$	$S^2 = 1084.66666$

Difference in variance

$$F = \frac{1084.66666}{866.45666} = 1.25$$

$$F_{.05}(24, 24) = 1.98$$

At the 5% significance level there is no reason to believe that there is any difference in the variance of the two mix designs.

Difference in means

$$t = \frac{\bar{Y}_1 - \bar{Y}_2}{\sqrt{\left[\frac{(N_1-1)S_1^2 + (N_2-1)S_2^2}{N_1 + N_2 - 2}\right]\left[\frac{1}{N_1} + \frac{1}{N_2}\right]}}$$

$$t = 56.60$$

$$t_{.10}(48) = 2.68 < 56.60$$

At the 5% significance level there is reason to believe that the mean of the HL mix is significantly larger than the mean of the LL mix.

Table 23

TEST FOR DIFFERENCE IN BATCH STRENGTHS

BEFORE AND AFTER FATIGUE TESTING - BATCH LL 1

Raw Strength

$Y_i = X_i$

Coded Strength

$$Y_i = \frac{X_i - 3200}{10}$$

Before Fatigue Testing	After Fatigue Testing		Before Fatigue Testing	After Fatigue Testing
3500	3270		+30	+ 7
3580	3250		+38	+ 5
3110	3470		- 9	+27
3560	3400		+36	+20
3570	3020		+55	-18
		N_j	5	5
		$\sum_i Y_{ij}$	150	41
		$\sum_i Y_{ij}^2$	6746	1527
		$\bar{Y}_j$	30.0	8.2
		s_j^2	561.50	336.75

$$F = \frac{561.50}{336.75} = 1.67$$

$$F_{.025}(4,4) = 9.60 > 1.67$$

At the 5% significance level there is no reason to believe that the two variances are not equal.

$$t = \frac{\bar{Y}_1 - \bar{Y}_2}{\sqrt{\left[\frac{(N_1 - 1)s_1^2 + (N_2-1)s_2^2}{N_1 + N_2 - 2}\right]\left[\frac{1}{N_1}+\frac{1}{N_2}\right]}}$$

$$t = 2.373$$

$$t_{.05}(8) = 2.306 < 2.373.$$

At the 5% significance level there is reason to believe that the two means are different. The data indicate that the strength decreases after fatigue testing.

Table 24

TEST FOR DIFFERENCE IN BATCH STRENGTHS

BEFORE AND AFTER FATIGUE TESTING

Batch LL 3

Raw Strength

$Y_i = X_i$

Coded Strength

$$Y_i = \frac{X_i - 3800}{10}$$

Before Fatigue Testing	After Fatigue Testing
3920	3970
4060	4060
4240	3810
4200	3850
3890	3800

$F = \frac{251.2}{127.7} = 1.97$

$F_{.025}(4, 4) = 9.60 > 1.97$

	Before Fatigue Testing	After Fatigue Testing
	12	17
	26	26
	44	1
	40	5
	9	0
N_j	5	5
$\sum_i Y_{ij}$	131	49
$\sum_i Y_{ij}^2$	4437	991
$\bar{Y}_j$	26.2	9.8
S_j^2	251.20	127.70

At the 5% significance level there is no reason to believe that the two variances differ.

$$t = \frac{\bar{Y}_1 - \bar{Y}_2}{\sqrt{\left[\frac{(N_1-1)\,S_1^2 + (N_2-1)S_2^2}{N_1 + N_2}\right]\left[\frac{1}{N_1} + \frac{1}{N_2}\right]}}$$

$t = 1.88$

$t_{.05}(8) = 2.306 > 1.88$

At the 5% significance level there is no reason to believe that the means are different.

Table 25

TEST FOR DIFFERENCE IN BATCH STRENGTHS

BEFORE AND AFTER FATIGUE TESTING

Batch HL 4

Raw Strength: $Y_i = X_i$

Coded Strength: $Y_i = \frac{X_i - 6250}{10}$

Raw: Before Fatigue Testing	Raw: After Fatigue Testing		Coded: Before Fatigue Testing	Coded: After Fatigue Testing
6260	5830		+ 1	-42
6250	6310		0	+ 6
6770	6340		+52	+ 9
6250	6280		0	+ 3
6250			0	
		N_j	5	4
		$\sum_i Y_{ij}$	53	-24
		$\sum_i Y_{ij}^2$	2705	3024
		$\bar{Y}_j$	10.6	- 6.0
		S_j^2	535.8	960.0

$$F = \frac{960.0}{535.8} = 1.79$$

$$F_{.025}(3, 4) = 9.98$$

At the 5% significance level there is no reason to believe that the two variances differ.

$$t = \frac{\bar{Y}_2 - \bar{Y}_1}{\sqrt{\left[\frac{(N_1-1)S_1^2 + (N_2-1)S_2^2}{N_1 + N_2 - 2}\right]\left[\frac{1}{N_1} + \frac{1}{N_2}\right]}}$$

$t = 0.924$

$t_{.05}(7) = 2.365 > 0.924.$

At the 5% significance level there is no reason to believe that the means are different.

Table 26

TEST FOR DIFFERENCE IN BATCH STRENGTHS

BEFORE AND AFTER FATIGUE TESTING

Batch HL 6

Raw Strength: $Y_i = X_i$

Coded Strength: $Y_i = \dfrac{X_i - 5750}{10}$

Raw: Before Fatigue Testing	Raw: After Fatigue Testing		Coded: Before Fatigue Testing	Coded: After Fatigue Testing
6000	5720		+25	- 3
6270	6100		+52	+35
6260	6040		+51	+29
5440	5280		-31	-47
6260	5640		+51	-11
$F = 1.16$		N_j	5	5
		$\sum_i Y_{ij}$	148	3
$F_{.025}(4,4) = 9.60 > 1.16$		$\sum_i Y_{ij}^2$	9492	4405
		$\bar{Y}_j$	29.6	0.6
		S_j^2	1277.8	1100.8

At the 5% significance level there is no reason to believe that the two variances are different.

$$t = \frac{\bar{Y}_1 - \bar{Y}_2}{\sqrt{\left[\dfrac{(N_1-1)S_1^2 + (N_2-1)S_2^2}{N_1 + N_2 - 2}\right]\left[\dfrac{1}{N_1} + \dfrac{1}{N_2}\right]}}$$

$t = 1.330$

$t_{.05}(8) = 2.306 > 1.330$

At the 5% significance level there is no reason to believe that the two means are different.

APPENDIX C

STATISTICAL ANALYSIS OF FATIGUE TEST DATA

Table 27

CALCULATIONS FOR RUN TEST

80% Stress Level

HL	5400,	6100,	6600,	86,300	110,000
LL	1600,	1900,	3500,	8,600,	19,100

Runs LL LL LL HL HL HL LL LL HL HL

$\mu = 4 \qquad 2 \leq \mu \leq 9$

70% Stress Level

HL	150,900,	166,100,	611,900,	733,600	
LL	26,300	43,000	51,400,	52,800,	105,100

Runs LL LL LL LL LL HL HL HL HL

$\mu = 2 \qquad 2 \leq \mu \leq 8$

60% Stress Level

HL	720,500,	736,400,	4,116,700,	6,737,500,	9,673,500 → (*)
LL	396,600,	688,700,	1,217,000,	1,610,600,	2,262,500

Runs LL LL HL HL LL LL LL HL HL

$\mu = 4 \qquad 2 \leq \mu \leq 8$

50% Stress Level

HL 4,751,300, 5,957,200, 10,474,400,→ 10,499,400,→
10,620,300,→ 12,196,100→

LL 3,147,600, 4,628,100, 4,926,400 5,673,300,
9,204,100 →

Runs LL LL HL LL LL HL

$\mu = 4 \qquad 4 \leq 5$

40% Stress Level

HL 10,216,500,→ 10,458,700 →

LL 10,005,400,→ 10,304,600,→ 10,418,100,→ 10,464,100 →
11,723,300,→

No run test can be made

(*) → indicates that the specimen did not fail. No run test can be made using these specimens since the actual number of cycles which will cause failure is unknown.

At the 5% significance level there is no reason to believe that the fatigue life populations of the two mixes differ at any of the stress levels tested.

Table 28

LINEAR REGRESSION CALCULATIONS

LOW-STRENGTH CONCRETE - ALL BATCHES

	Stress Level - X_j		
	60%	70%	80%
Log of cycles to failure - Y_i	5.83803	4.63347	4.28103
	5.59835	4.72263	3.54407
	6.20699	4.71096	3.20412
	6.08529	5.02160	3.93450
	6.35459	4.41996	3.27875
N_j	5	5	5
$\sum_i Y_{ij}$	30.08325	23.50862	18.24247
$\sum_i Y_{ij}^2$	181.36241	110.71794	67.38453
$(\sum_i Y_{ij})^2/N_j$	181.00038	110.53104	66.55754
$\bar{Y}_j$	6.01665	4.70172	3.34849

$$SS_{xy} = \sum_j \sum_i X_j Y_{ij} - \frac{\sum_j \sum_i Y_{ij}}{\sum_j N_j} (\sum_j N_j X_j) = -118.40150$$

$$SS_x = \sum_j N_j X_j^2 - \frac{1}{\sum_j N_j} (\sum N_j X_j)^2 = 1000$$

$$b = \frac{SS_{xy}}{SS_x} = -0.11840$$

$$a = \sum_j \sum_i Y_{ij} / \sum_j N_j = 4.78895$$

$$\bar{X} = \sum_j N_j X_j / \sum_j N_j = 70$$

$$\hat{y} = a + b(X - \bar{X})$$

where y = log of the cycles to failure and X = the stress level.

Table 28 (continued)

$$\widehat{\log N} = 13.07695 - 0.11840\ S$$

$$SS_l = \sum_j \frac{(\sum_i Y_{ij})^2}{N_j} - \frac{(\sum_j \sum_i Y_{ij})^2}{\sum N_j} = 14.07747$$

$$SS_e = \sum_j \sum_i Y_{ij}^2 - \sum_j \frac{(\sum_i Y_{ij})^2}{N_j} = 1.37592$$

$$SS_t = \sum_j \sum_i Y_{ij}^2 - \frac{(\sum_j \sum_i Y_{ij})^2}{\sum_j N_j} = 15.45339$$

ANOVA TABLE

Source	df	Sum of Squares	Mean Square	F ratio
Stress levels	2	14.07747	7.03873	
Regression	1	14.01856	14.01856	
Departure	1	0.05891	0.05891	0.51
Error	12	1.37592	0.11466	
Total	14	15.45339		

$$r^2 = \frac{\text{Regression SS}}{\text{Total SS}} = 0.90715$$

$$\eta^2 = \frac{\text{Level SS}}{\text{Total SS}} = 0.91096$$

$$r = 0.95244$$

$$F_{.05}(1,12) = 4.75 > 0.51$$

At the 5% significance level there is no reason to believe that the S-N relationship is not linear.

Table 29

LINEAR REGRESSION CALCULATIONS

HIGH-STRENGTH CONCRETE

Batches HL 2, HL 3, HL 4

Stress Level - X_j

	60%	70%	80%
	5.85763 5.86711	5.17869 5.22037	3.78533 3.81954 3.73239
N_j	2	2	3
$\sum_i Y_{ij}$	11.72474	10.39906	11.33726
$\sum_i Y_{ij}^2$	68.73481	54.07109	42.84834
$(\sum_i Y_{ij})^2 / N_j$	68.73476	54.07022	42.84448
$\bar{Y}_j$	5.86237	5.19953	3.77908

$$SS_{xy} = \sum_j \sum_i X_j Y_{ij} - \frac{\sum_j \sum_i Y_{ij}}{\sum_j N_j} \left(\sum_j N_j X_j \right) = -51.67560$$

$$SS_x = \sum_j N_j X_j^2 - \frac{1}{\sum_j N_j} \left(\sum N_j X_j \right)^2 = 485.71429$$

$$b = \frac{SS_{xy}}{SS_x} = -0.10639$$

$$a = \sum_j \sum_i Y_{ij} / \sum_j N_j = 4.78015$$

$$\bar{X} = \sum_j N_j X_j / \sum_j N_j = 71.42857$$

Table 29 (continued)

$$\widehat{Y} = a + b\,(X - \bar{X})$$

where

Y = log of the cycles to failure

and X = the stress level

$$\widehat{\text{Log } N} = 12.37944 - 0.10639\,S$$

$$SS_1 = \sum_j \frac{(\sum_i Y_{ij})^2}{N_j} - \frac{(\sum_j \sum_i Y_{ij})^2}{\sum_j N_j} = 5.70053$$

$$SS_e = \sum_j \sum_i Y_{ij}^2 - \sum_j \frac{(\sum_i Y_{ij})^2}{N_j} = 0.00478$$

$$SS_t = \sum_j \sum_i Y_{ij}^2 - \frac{(\sum_j \sum_i Y_{ij})^2}{\sum_j N_j} = 5.70531$$

ANOVA TABLE

Source	df	Sum of Squares	Mean Square	F ratio
Stress levels	2	5.70053	2.85026	
Regression	1	5.49829	5.49829	
Departure	1	0.20224	0.20224	169.95
Error	4	0.00478	0.00119	
Total	6	5.70531		

$$r^2 = \frac{\text{Regression SS}}{\text{Total SS}} = 0.96371$$

$$\eta^2 = \frac{\text{Level SS}}{\text{Total SS}} = 0.99916$$

$$r = 0.98169$$

$$F_{.05}(1,4) = 7.71 < 169.95.$$

At the 5% significance level there is reason to believe that the S-N relationship is not linear.

Table 30

LINEAR REGRESSION CALCULATIONS

HIGH-STRENGTH CONCRETE

Batches HL 5, HL 6

	Stress Level - X_j		
	60%	70%	80%
	6.82850 6.61455	5.78669 5.87714	5.04139 4.93601
N_j	2	2	2
$\sum_i Y_{ij}$	13.44305	11.66382	9.97740
$\sum_i Y_{ij}^2$	90.38068	68.02644	49.77981
$(\sum_i Y_{ij})^2 / N_j$	90.35779	68.02235	49.77425
$\bar{Y}_j$	6.72152	5.83191	4.98870

$$SS_{xy} = \sum_j \sum_i X_j Y_{ij} - \frac{\sum_j \sum_i Y_{ij}}{\sum_j N_j} \left(\sum_j N_j X_j \right) = -33.95580$$

$$SS_c = \sum_j N_j X_j^2 - \frac{1}{\sum_j N_j} \left(\sum_j N_j X_j \right)^2 = 400$$

$$b = \frac{SS_{xy}}{SS_x} = -0.08488$$

$$a = \sum_j \sum_i Y_{ij} / \sum_j N_j = 5.84571$$

$$\bar{X} = \sum_j N_j X_j / \sum_j N_j = 70$$

$$\widehat{Y} = a + b (X - \bar{X})$$

where Y = log of the cycles to failure and X = the stress level

$\widehat{\log N}$ = 11.07695 - 0.08488 S

Table 30 (continued)

$$SS_l = \sum_j \frac{(\sum_i Y_{ij})^2}{N_j} - \frac{(\sum_j \sum_i Y_{ij})^2}{\sum_j N_j} = 3.12032$$

$$SS_e = \sum_j \sum_i Y_{ij}^2 - \sum_j \frac{(\sum_i Y_{ij})^2}{N_j} = 0.03254$$

$$SS_t = \sum_j \sum_i Y_{ij}^2 - \frac{(\sum_j \sum_i Y_{ij})^2}{\sum_j N_j} = 3.15288$$

ANOVA TABLE

Source	df	Sum of Squares	Mean Square	F ratio
Stress levels	2	3.12032	1.56016	
Regression	1	2.88184	2.88184	
Departure	1	0.23848	0.23848	22.00
Error	3	0.03254	0.01084	
Total	5	3.15286		

$$r^2 = \frac{\text{Regression SS}}{\text{Total SS}} = 0.91403$$

$$\eta^2 = \frac{\text{Level SS}}{\text{Total SS}} = 0.98967$$

$$r = 0.95605$$

$$F_{.05}(1,3) = 10.1 < 22.0$$

At the 5% significance level there is reason to believe that the S-N relationship is not linear.

Table 31

TEST FOR DIFFERENCE IN CORRELATION COEFFICIENTS

Sample	N-3	r	Z	(N-3)Z	$(N-3)Z^2$
I	12	0.95244	1.85741	22.28892	41.39964
II	4	0.98169	2.34213	9.36852	21.94228
III	3	0.95605	1.89781	5.69343	10.80504
	19			37.35087	74.14696

$$\text{Average } Z = \frac{\Sigma(N-3)Z}{\Sigma(N-3)} = 1.96583$$

$$\text{Average } Z \times \Sigma(N-3)Z = 73.42546$$

$$\chi^2 = 74.14696 - 73.42546 = 0.72150$$

$$\chi^2_{.05}(2) = 5.99 > 0.72$$

where

$$Z = 1.1513 \, [\log_{10}(1+r) - \log_{10}(1-r)]$$

At the 5% significance level there is no reason to believe that the three correlation coefficients are different.

Table 32

COMPARISON OF SLOPES

All of the following values can be found directly in the linear regression calculations and are only summarized in this table. The definitions of the symbols used are found in Table 107 of Reference 23.

$E_{yy_1} = 15.45339$	$E_{xy_1} = -118.40150$	$E_{xx_1} = 1000.00000$
$E_{yy_2} = 5.70531$	$E_{xy_2} = -51.67560$	$E_{xx_2} = 485.71429$
$E_{yy_3} = 3.15286$	$E_{xy_3} = -33.95580$	$E_{xx_3} = 400.00000$
$E_{yy} = 24.31156$	$E_{xy} = -204.03290$	$E_{xx} = 1885.71429$
$C_{y_1} = 71.83434$	$C^2_{y_1}/N_1 = 344.01149$	$K_y = 703.70158$
$C_{y_2} = 33.46106$	$C^2_{y_2}/N_2 = 159.94893$	
$C_{y_3} = \underline{35.07427}$ 140.36967 Total	$C^2_{y_3}/N_3 = \underline{205.03407}$ 708.99449 Total	$C_{yy} = 5.29291$
$C_{x_1} = 1050$	$C^2_{x_1}/N_1 = 73500.00000$	$K_x = 138703.57142$
$C_{x_2} = 500$	$C^2_{x_2}/N_2 = 35714.28571$	
$C_{x_3} = \underline{420}$ 1970 Total	$C^2_{x_3}/N_3 = \underline{29400.00000}$ 138714.28571 Total	$C_{xx} = 10.71429$

$C_{x_1y_1} / N_1 = 5028.40380$	$K_{xy} = 9876.00892$
$C_{x_2y_2} / N_2 = 2390.07571$	
$C_{x_3y_3} / N_3 = \underline{2455.19890}$ 9873.67841 Total	$C_{xy} = -\ 2.33051$
$\sum_j \sum_i Y^2_{ij} = 733.30605$	$S_{yy} = 29.60447$
$\sum_j \sum_i X^2_{ij} = 140500.00000$	$S_{xx} = 1896.42858$
$\sum_j \sum_i X_{ij} Y_{ij} = 9669.63780$	$S_{xy} = -\ 206.37112$
$\sum_j \dfrac{E^2_{xy_j}}{E_{xx_j}} = 22.39921$	$\dfrac{E^2_{xy}}{E_{xx}} = 22.07620$

Table 32 (continued)

$$\frac{C^2_{xy}}{C_{xx}} = 0.50691 \qquad \frac{S^2_{xy}}{S_{xx}} = 22.45749$$

General Model

$$Y_{ij} = \mu + \theta_j + B_m(\bar{X}_j - \bar{X}) + B_a\,(X_{ij} - \bar{X}_j) + \delta_j\,(X_{ij} - \bar{X}_j) + \epsilon_{ij}$$

ANALYSIS OF COVARIANCE TABLE

Source of variation	Sum of Squares	df	Mean Square
Deviation from regression within groups	1.91235	22	0.08692
Differences between regressions within groups	0.32301	2	0.16150
Deviations within classes from B_a	2.23536	24	0.09314
Deviations between groups from B_m	4.78600	1	4.78600
Difference between B_a and B_m	0.12562	1	0.12562
Common overall regression B_o	22.45749	1	22.45749
Total	29.60447	27	

Test for departure from a common over-all regression

$$S^2\text{over-all} = \frac{0.32301 + 9.78600 + 0.12562}{2 + 1 + 1}$$

$$= 1.30865$$

$$F = \frac{1.30865}{0.08692} = 15.056$$

$$F_{.005}(4,22) = 5.017 < 15.056$$

At the 5% significance level there is reason to believe that the three slopes deviate from a common over-all slope.

Table 32 (continued)

Test for difference in slopes

$$F = \frac{0.16150}{0.08692} = 1.858$$

$$F_{.005}(2,22) = 6.806 < 1.858$$

At the 5% significance level there is no reason to believe that the three slopes are different.

Table 33

TEST FOR DIFFERENCES BETWEEN INTERCEPTS

This test is a two-way analysis of variance in which the sums of squares have been calculated from the linear regression equations

ANOVA TABLE

Source	df	Sum of Squares	Mean Square	F-ratio
Samples	2	5.19265	2.59632	34.9
Stress levels	2	22.61248	11.30624	
Interaction	4	0.28584	0.07146	
Error	19	1.41324	0.07438	
Total	27	29.50421		

$$F_{.005}(2,19) = 7.094 < 34.9$$

At the 5% significance level there is reason to believe that the intercepts are not equal.

$$\bar{Y}_1 = 4.78895 \qquad S_{\bar{y}} = \sqrt{\frac{0.07438}{2}}$$

$$\bar{Y}_2 = 4.78015$$

$$\bar{Y}_3 = 5.84571 \qquad = 0.1928$$

$$R_3 = 0.691 \qquad R_2 = 0.570$$

Difference Table

Sample	2	1
3	1.06556	1.05676
1	0.00880	

At the 5% significance level there is reason to believe that the intercept of sample III is different than the intercepts of the other two samples.

Table 34

CALCULATION FOR PREDICTION INTERVALS

Equation for prediction interval

$$L = y \pm T_{.05}(N-2)\, S_{yx} \sqrt{\frac{1}{N} + \frac{(X_j - \bar{X})^2}{SS_x}}$$

where

$$S_{yx} = \sqrt{\frac{\text{Departure SS} \quad \text{Error SS}}{N - 2}}$$

Low-Strength Concrete

$S_{yx} = 0.33222$ $\qquad N = 15$

$SS_x = 1000$ $\qquad T_{.05}(13) = 2.1064$

$X_j = 60\%$

$L_1 = 5.97295 - 0.29301 = 5.67994$
$L_u = 5.97295 + 0.29301 = 6.26596$

$X_j = 70\%$

$L_1 = 4.78895 - 0.18532 = 4.60363$
$L_u = 4.78895 + 0.18532 = 4.97427$

$X_j = 80\%$

$L_1 = 3.60495 - 0.29301 = 3.31194$
$L_u = 3.60495 + 0.29301 = 3.89796$

High-Strength Concrete

$S_{yx} = 0.20349$ $\qquad N = 7$

$SS_x = 485.41429$ $\qquad t_{.05}(5) = 2.5706$

$X_j = 60\%$

$L_1 = 5.99604 - 0.33566 = 5.66038$
$L_u = 5.99604 + 0.33566 = 6.33170$

$X_j = 70\%$

$L_1 = 4.93214 - 0.20059 = 4.73155$
$L_u = 4.93214 + 0.20059 = 5.13273$

$X_j = 80\%$

$L_1 = 3.86824 - 0.28368 = 3.58456$
$L_u = 3.86824 + 0.28368 = 4.15192$

Table 34 (continued)

High-Strength Concrete

$S_{yx} = 0.26029$	$N = 6$		
$SS_x = 400$	$t_{.05}(4) = 2.7789$		
$X_j = 60\%$	$L_1 = 6.69451 - 0.46647$	$= 6.22804$	
	$L_u = 6.69451 + 0.46647$	$= 7.16098$	
$X_j = 70\%$	$L_1 = 5.84571 - 0.29502$	$= 5.59065$	
	$L_u = 5.84571 + 0.29502$	$= 6.14073$	
$X_j = 80\%$	$L_1 = 4.99691 - 0.46647$	$= 4.53044$	
	$L_u = 4.99691 + 0.46647$	$= 5.46338$	

Table 35

TEST TO DETERMINE DIFFERENCE IN FATIGUE LIFE

WHEN TESTING AT DIFFERENT SPEEDS

<u>Coding</u>: $Y_i = \frac{X_i}{1000}$

where

X_i is the number of cycles to cause failure of specimens from batch HL 1 which were tested at the 80% stress level.

The average batch strength was 5,130 psi.

<u>Testing Machine</u>

Amsler 500 cycles/minute	Krouse-Purdue 1,000 cycles/minute	
33.7	45.4	
17.2	9.7	
1.1	3.2	
20.8	7.8	
5.8	6.3	
71.7	33.3	
13.6	23.4	
4.8	23.2	
86.4	145.4	
9	9	N_j
255.1	297.7	$\sum_i Y_{ij}$
14712.87	25601.87	$\sum_i Y_{ij}^2$
28.34	30.08	$\bar{Y}_j$
935.55	1969.46	s_j^2

$F = 2.015$

$F_{.05}(8,8) = 3.44 > 2.015$

$t = 0.264$

$t_{.05}(16) = 2.12 > 0.264$

At the 5% significance level there is no reason to believe that there is any difference in the fatigue life of lightweight aggregate concrete when it is tested at speeds of 500 cycles per minute and 1000 cycles per minute.

APPENDIX D

STATISTICAL COMPARISON OF LIGHTWEIGHT CONCRETE WITH NORMAL WEIGHT CONCRETE

Table 36

TEST FOR DIFFERENCE IN CORRELATION COEFFICIENTS

Sample	N-3	r	Z	(N-3)Z	$(N-3)Z^2$
I	12	0.95241	1.85741	22.28892	41.39964
II	4	0.98169	2.34213	9.36852	21.94228
III	3	0.95605	1.89781	5.69343	10.80504
FN	13	0.61863	0.72280	9.39640	6.79172
FA	13	0.93632	1.70735	22.19555	37.89557
	45			68.94282	118.83425

$$\text{Average } Z = \frac{\Sigma(N-3)Z}{\Sigma(N-3)} = 1.53206$$

$$\text{Average } Z \times (N-3)Z = 105.62454$$

$$\chi^2 = 118.83425 - 105.62454 = 13.20971$$

$$\chi^2_{.05}(4) = 9.49$$

At the 5% significance level there is reason to believe that there is a difference between the five correlation coefficients.

Table 37

COMPARISON OF SLOPES

The values summarized in this Table can be found from Appendix C of Reference (22) and Table 31 of Appendix C of this thesis.

$E_{yy_{FN}} = 13.41620$	$E_{xy_{FN}} = -45.62244$	$E_{xx_{FN}} = 413.75000$
$E_{yy_{FA}} = 22.03059$	$E_{xy_{FA}} = -90.24373$	$E_{xx_{FA}} = 421.75000$
$E_{yy} = 59.75835$	$E_{xy} = -340.90907$	$E_{xx} = 2721.21429$

$C_{y_{FN}} = 69.35908$	$C^2_{y_{FN}}/N_{FN} = 300.66762$	$K_y = 1279.76178$
$C_{y_{FA}} = 67.37359$	$C^2_{y_{FA}}/N_{FA} = 283.70003$	$C_{yy} = 13.60036$
$C_{x_{FN}} = 1194$	$C^2_{x_{FN}}/N_{FN} = 89102.250$	$K_x = 320032.06666$
$C_{x_{FA}} = 1218$	$C^2_{x_{FA}}/N_{FA} = 92720.250$	$C_{xx} = 404.71905$

$\dfrac{C_{x_{FN}}\,Y_{FN}}{N_{FN}} = 5175.92134$	$K_{xy} = 20237.69734$
$\dfrac{C_{x_{FA}}\,Y_{FA}}{N_{FA}} = 5128.81453$	$C_{xy} = -59.28306$
$\sum_j \sum_i Y_{ij}^2 = 1353.12050$	$S_{yy} = 73.35872$
$\sum_j \sum_i X_{ij}^2 = 323158.00000$	$S_{xx} = 3125.93334$
$\sum_j \sum_i X_{ij}\,Y_{ij} = 19837.49751$	$S_{xy} = -400.19983$

Table 37 (continued)

ANALYSIS OF COVARIANCE TABLE

Source of variation	Sum of Squares	df	Mean Square
Deviation from regression within groups	12.77316	50	0.25546
Difference between regressions within groups	4.25837	4	1.96459
Deviation within classes from B_a	17.03153	54	0.31539
Deviations between groups from B_m	4.91661	3	1.63887
Difference between B_a and B_m	0.15639	1	0.15639
Common over-all regression B_o	51.23586	1	51.23586
Total	73.35872	59	

Test for departure from a common over-all regression

$$S^2 \text{ over-all} = \frac{4.25837 + 4.91661 + 0.15639}{4 + 3 + 1}$$

$$= 1.16639$$

$$F = \frac{1.16639}{0.25546} = 4.56595$$

$$F_{.005}(8,50) = 3.24 < 4.57$$

At the 5% significance level there is reason to believe that there is a departure from an over-all regression.

Table 37 (continued)

Test for difference between slopes

$$F = \frac{1.06459}{0.25546} = 4.16734$$

$$F_{.005}(4,\ 50) = 4.26 > 4.17$$

At the 5% significance level there is no reason to believe that there is any difference between the five slopes.